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PNNL-33001

Total Factor Productivity Studies in the Electricity Sector

An Overview of Methodologies and Best Practices

June 2022

Brittany Tarufelli, PhD and Mark Weimar, PhD
Pacific Northwest National Laboratory

U.S. DEPARTMENT OF ENERGY

Prepared for the U.S. Department of Energy
under Contract DE-AC05-76RL01830

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BATTELLE
for the
UNITED STATES DEPARTMENT OF ENERGY
under Contract DE-AC05-76RL01830

Printed in the United States of America

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Summary

Performance-based regulation was introduced to improve upon and as an alternative to cost-of-service regulation, providing utilities with incentives similar to those faced by companies operating in competitive markets, and encouraging them to focus on operational efficiency and cost reductions. Although there are many approaches to strengthen utility performance incentives, in this review we primarily focus on total factor productivity (TFP) studies that inform the X-factor in price or revenue cap regulation.

Under price or revenue cap regulation, prices or revenues are typically indexed to a macroeconomic inflation indicator and reduced by a productivity offset, the X-factor. The objective of the X-factor is to adjust the macroeconomic inflation indicator to fit the needs of the electric industry. When benchmarked to the economy, the X-factor sums the difference in TFP growth rates between the electric industry and the rest of the economy and the difference in input price growth rates between the rest of the economy and the electric industry.

TFP is simply the difference in growth rates between a company's physical outputs and physical inputs. A more productive firm will be able to produce more outputs given the same level of inputs than a less productive firm. There are many approaches to estimating TFP including frontier, non-frontier, parametric, and non-parametric methods. In this review, we summarize the most common approaches used in TFP studies, focusing on index number methodologies which are the preferred approach for estimating TFP in performance-based regulation in North America.

Key challenges in TFP measurement include the measurement of output, the measurement of input—especially the concept of capital—missing or inappropriate data, and the weights used for indexes. To address these challenges, we provide a review of common methods for measuring and aggregating outputs and inputs for the electricity sector. Output is typically measured as a combination of demand- and supply-side factors, although experts vary in their choice of which measures to include. We provide an overview of common output measurement approaches and potential bias, concluding that output indexes can consist of more than one output measure to incorporate both customer- and sales-density variables for measuring output for TFP analysis.

We also provide an overview of common input measurement approaches, including an in-depth guide to the depreciation assumptions and models used to determine capital quantity and cost. We conclude that although there are multiple depreciation assumptions and models that may be appropriate for valuing capital, the depreciation assumption should best reflect the underlying depreciation profile of the asset, capital quantity and price indexes should be consistent (i.e., reflect the same depreciation assumptions), and sensitivity analyses can be performed to determine impacts to TFP from using different depreciation assumptions.

From our review of the literature and best practices of North American performance-based regulation, we provide several recommendations regarding the methodology and assumptions underlying TFP studies, including that study methodologies and assumptions should be transparent enough that the study could be reproduced, and sensitivity analysis of key assumptions should be undertaken. Although there are a variety of acceptable assumptions that can be employed in a TFP study, depending on the underlying data and unique circumstances for each utility, we provide an overview of potential biases from varied assumptions as well as recommendations to improve TFP estimation.

Acronyms and Abbreviations

AUC	Alberta Utilities Commission
CCA	Consumer Choice Advocate
DEA	Data envelopment analysis
DOE	Department of Energy
FERC	Federal Energy Regulatory Commission
FTE	Full-time equivalent employees
GDP	Gross domestic product
GDPPI	Gross domestic product price index
GMM	Generalized method of moments
M&S	Materials and services
MRS	Materials, rents and services
O&M	Operations and maintenance
OM&A	Operations, maintenance, and administration
OLS	Ordinary least squares
PBR	Performance-based regulation
TFP	Total factor productivity

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1.0 Introduction

Performance-based regulation was introduced as an improvement and as an alternative to cost-of-service regulation by providing incentives for operational efficiency and cost reductions. Performance-based regulation provides utilities with incentives like those faced by companies operating in competitive markets, encouraging them to focus on reducing costs and improving other performance dimensions.

Price or revenue cap regulation provides strong incentives for cost reduction by specifying the rate at which inflation-adjusted prices or revenues must decline.[1] By decoupling prices or revenues from a company's production costs, price or revenue cap regulation encourages companies to reap financial benefits through reducing their operating costs. As discussed in this report, the specified rate at which inflation-adjusted prices or revenues must decline is known as the X-factor. The X-factor reflects the extent to which: 1) the growth rate in total factor productivity (TFP) of the regulated utility industry exceeds that of the entire economy, and 2) the prices of inputs used by regulated utilities rise less quickly than the prices of inputs used by other companies in the economy (Bernstein and Sappington 1999).

Although performance-based regulation may include many features, such as price or revenue caps, rate freezes, and earnings-sharing mechanisms, in this review we will primarily focus on TFP studies that inform the X-factor in price or revenue cap regulation. This report provides an overview of the economic principles of the X-factor, methodologies for estimating TFP – including index number methods commonly used in performance-based regulation, common approaches to measuring outputs and inputs for TFP studies using index number methods, a review of lessons learned from the TFP literature and recent performance-based regulation cases, and a summary of factors which may bias TFP studies and recommendations to address potential biases.

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[1] Recent TFP trends in electric distribution have shown lower productivity growth, leading to negative X-factors, which can increase inflation-adjusted prices or revenues. Negative TFP trends can be driven by both slower growth in output and faster growth in inputs (Meitzen et al. 2018).

2.0 Economic Principles of the X-Factor

Bernstein and Sappington (1999) show that in a competitive market (with long-term profits driven to zero) that a company's growth rate in output prices, $\dot{P}$, is equal to its input price growth rate, $\dot{W}$, minus its TFP growth rate, $\dot{T}$—equivalently,

$$\dot{P} = \dot{W} - \dot{T}, \tag{1}$$

where the company's TFP growth rate is simply the growth rate of its output minus the growth rate of its input quantities. However, if regulators were to adjust output prices based on a company's past input and productivity, the results would not improve upon cost-of-service regulation. Instead, regulators could forecast future input prices and productivity, allowing output prices to vary with projected changes in these variables, encouraging a company to improve performance beyond expectations. The longer the timeframe in which a company's approved prices are decoupled from its production costs, the stronger the incentives the company faces to reduce its production costs and enhance productivity.

However, strong performance incentives depend on accurately forecasting changes in input prices and TFP. Another option is to benchmark the electricity industry to the rest of the economy, to recreate the pressures of a competitive market (Bernstein and Sappington, 1999). Subtracting the expected economy-wide increases in output prices, $\dot{P}^E$, due to economy-wide increases in input prices, $\dot{W}^E$, minus economy-wide increases in productivity, $\dot{T}^E$, $\dot{P}^E = \dot{W}^E - \dot{T}^E$, from Equation (1) yields

$$\dot{P} = \dot{P}^E - \left[\dot{T} - \dot{T}^E\right] + \left[\dot{W}^E - \dot{W}\right], \tag{2}$$

where the X-factor is equal to

$$X = \left[\dot{T} - \dot{T}^E\right] + \left[\dot{W}^E - \dot{W}\right]. \tag{3}$$

In practice, this is equivalent to letting output rise at the rate of inflation, $\dot{P}^E$, minus an offset, where the offset is the X-factor, X. The X-factor sums the difference in TFP growth rates in the electric industry and the rest of the economy $\left[\dot{T} - \dot{T}^E\right]$ (TFP differential) and the difference in input price growth rates between the rest of the economy and the electric industry $\left[\dot{W}^E - \dot{W}\right]$ (input price differential) (Bernstein and Sappington 1999).[1]

Relating the economic principles of the X-factor to price and revenue caps, under price cap regulation, average prices are typically indexed to some macroeconomic inflation indicator (i.e., the consumer price index [CPI], the producer price index [PPI], or the chain-weighted price index for gross domestic product [GDPPI]) and reduced by the productivity offset, or X-factor, such that $ln(P) = GDPPI - X$, where the natural logarithm approximates the growth rate. Revenue caps are similar, except that price (P) is replaced by revenue (R) and the formula is $ln(R) = GDPPI - X$. Price-cap regulated firms are only allowed to adjust their quantities as prices are set according to the price cap index, whereas revenue-cap regulated firms are allowed to adjust both prices and quantities—as long as the revenue cap index is not exceeded. The choice of a price or revenue cap also affects a firm's exposure to risk. With a price cap, a

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[1] If an industry input inflation measure is used instead of an economy-wide output inflation indicator, the X factor is instead $X = \dot{T}$. See Meitzen et al. (2017) for further reading.

firm is allowed to set rates in accordance with the approved index, and the firm is subject to volumetric risk (if demand declines, the firm earns less revenue). With a revenue cap, a firm may change customer rates, so long as the revenue does not exceed the revenue cap, protecting a firm from volumetric risk. This difference in incentives provided by price vs. revenue caps has led more recent performance-based regulation in the electricity sector to focus on revenue cap regulation, which does not incentivize demand growth and is more in line with conservation efforts (Weisman 2018).[1]

Although the focus of this review is on the TFP study that informs a portion of the X-factor, another important consideration is the choice of inflation indicator. The X-factor shown in equation (3) benchmarks electricity industry performance to the rest of the economy. When a macroeconomic inflation index such as the GDPPI is used to measure inflation, there is an additional term, $[\dot{W}^E - \dot{W}]$, known as the input price or inflation differential, that must also be estimated. If the input price trend of the economy rises more rapidly than that of the electric industry, the X-factor will be larger, slowing price or revenue growth.

Other factors, such as possible developments that are outside of a firm's control such as changes in tax rates or other government policies that are not reflected in inflation or X-factors and can differ from period to period are often captured in a *Z*-factor, i.e., $= GDPPI - X \pm Z$. A stretch factor, *S*, can be included to allow for the increased productivity growth a utility may experience in the change from traditional cost-of-service regulation to performance-based regulation, i.e., $= GDPPI - X - S \pm Z$. Last, a supplemental capital factor, *K*, may be included if it is believed that sufficient revenues will not be generated to finance infrastructure investments, i.e., $= GDPPI - X - S \pm Z + K$. Earnings-sharing mechanisms may also be included to share excess earnings with customers. See Meitzen et al. (2017) for further reading.

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[1] Another option is a revenue-per-customer cap, which allows revenue to grow if the number of customers grows. Under a revenue cap the regulated firm may bear the financial risk associated with customer growth (Weisman 2018).

3.0 Estimating Total Factor Productivity

Allowing for an acceptable rise in price or revenue in performance-based regulation requires estimating TFP. TFP is simply the difference in growth rates between a company's physical outputs and physical inputs. A more productive firm will be able to produce more outputs given the same level of inputs than a less productive firm.

When estimating TFP, it is necessary to observe inputs and outputs accurately as well as to control for the potential input substitution that a firm's production technology allows. To make this idea more concrete, following Van Biesebroeck (2007), a simple model of production with a single output will be used to explain how different methodologies account for a firm's production technology and control for the possibility of input substitution. Let Q represent the single output produced by firm *i* at time *t,* let *X* represent the vector of inputs used in the production process, and let $F(\cdot)$ represent the production technology. Let A represent differences in productivity both among firms and over time. A is typically unobservable, and thus it is the parameter TFP studies aim to estimate,

$$Q_{it} = A_{it}F_{it}(X_{it}). \tag{4}$$

Allowing for the possibility that technology can vary across firms requires specifying which production technology underlies the comparison of firms. By taking logarithms and rearranging the production function, and by letting the subscript $k \in \{it, j\tau\}$ specify which production technology underlies the comparison of firms, one can compare how much extra output firm i produces at time t relative to firm j at time τ, conditional on their use of inputs (X_{it} and $X_{j\tau}$, respectively) using production technology $F_k(\cdot)$, by evaluating:

$$ln\left(\frac{A_{it}}{A_{j\tau}}\right)_k = ln\left(\frac{Q_{it}}{Q_{j\tau}}\right) - ln\left(\frac{F_k(X_{it})}{F_k(X_{j\tau})}\right). \tag{5}$$

In practice, productivity comparisons across multiple firms in the same industry can be achieved if average productivity across all firms is used in the denominator—equivalent to subtracting the logarithm of the arithmetic mean of the average firm from the logarithm of the individual firm, $ln(A_{it}) - \overline{ln(A_t)}$, yielding the percentage change in TFP for the individual firm compared to the average firm. Different methodologies then differ in how the last term—the ratio of aggregated input—is determined (Van Biesebroeck 2007).

Methodologies can be frontier or non-frontier, where frontier approaches define the most efficient (or best practice) firms using a bounding function or a set of best obtainable positions (Mahadeven 2003). Frontier approaches differ from non-frontier approaches in that they can differentiate the role of technical efficiency, or movements toward the production frontier, from technological progress, or outward shifts in the production function (production technology). Non-frontier approaches assume all firms are technically efficient, and any changes in TFP growth are due to technological change. Another difference is that frontier approaches allow a firm to be benchmarked to the most efficient firm in the sample, whereas non-frontier approaches often compare a firm to the average firm using statistical techniques (Mahadeven 2003). Lawrence and Diewert (2004) find that the non-frontier approach replicates the market outcome more closely but runs the risk of too low of a target being set for a firm. Frontier approaches, on the other hand, can set a high target for a firm, but that target may be unrealistic if there are data errors, as these methods are more sensitive to data errors and omissions.

Both frontier and non-frontier approaches can be estimated parametrically or non-parametrically. Parametric methods require a specific functional form for the production technology, with parameters estimated empirically using econometrics on the sample data for inputs and outputs. Parametric methods are sensitive to the functional form and results can change when the functional form is changed. Non-parametric methods do not impose any functional forms but have the drawback that they cannot be validated by statistical tests (Mahadeven, 2003).

Table 1 summarizes common approaches used in TFP studies, with index number methodologies being the preferred approach in performance-based regulation in North America. Non-frontier methods will be discussed in more detail in the subsequent section. Frontier methods, more commonly used for benchmarking analysis, are discussed in Appendix A for the interested reader.

Table 1. Common Approaches to Determine Total Factor Productivity

	Non-Frontier	Frontier
Non-Parametric	Index Number Methods	Data Envelopment Analysis
Parametric	Ordinary Least Squares and Other Econometric Methods	Stochastic Frontier Methods

3.1 Non-Parametric Methods: Index Numbers

Index number methods combine changes in diverse outputs and inputs into measures of change in total outputs and total inputs. These methodologies essentially take a weighted average of the changes in outputs and inputs. Different index number methodologies differ in how they take the weighted average (Lawrence and Diewert 2004).[1]

Performance-based regulation index number approaches typically follow an economic approach to estimating TFP. This approach allows researchers to relate properties of index numbers to properties of production functions, providing a link between economic theories of production and index number methodologies—an approach pioneered by Solow (1957). A brief history of the evolution of the economic approach to estimating TFP will be discussed in order to provide context to the reasons why certain index number methodologies are favored in TFP studies for performance-based regulation.

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[1] Index number methodologies assume that firms make optimal input choices, but different methods restrict how the underlying production technology may differ among firms. The index number approach uses information from input factor price ratios to approximate the slope of the production function (which represents production technology, F[·]) with a Taylor series expansion (used to approximate a function's value in terms of a function's derivatives at a specific point as the function and the sum of its derivatives are equal near this point), providing a theoretically grounded aggregation method for inputs and outputs while not requiring the researcher to know the exact shape of the production technology (Hulten 2001; Van Beisebroeck 2007).

Solow (1957) defined an aggregate production function where output, Q_{it}, is a function of labor, L_{it}, capital, K_{it}, and technical change, A_{it}, for firm i at time t. Importantly, he assumed technical change was neutral, allowing for increases or decreases in outputs to be determined by a shift in the production function (production technology),

$$Q_{it} = A_{it} F(L_{it}, K_{it}). \tag{6}$$

By totally differentiating the above equation with respect to time and dividing by Q_{it}, Solow (1957) showed that growth in real output can be factored into the growth of capital and labor, weighted by their output elasticities, and the growth rate in the technical change parameter, as shown in the following equation:

$$\frac{\dot{Q}_{it}}{Q_{it}} = \frac{\partial Q}{\partial L}\frac{L_{it}}{Q_{it}}\frac{\dot{L}_{it}}{L_{it}} + \frac{\partial Q}{\partial K}\frac{K_{it}}{Q_{it}}\frac{\dot{K}_{it}}{K_{it}} + \frac{\dot{A}_{it}}{A_{it}}, \tag{7}$$

where dots indicate time derivatives. Letting $s_{it}^{L} = \frac{\partial Q}{\partial L}\frac{L_{it}}{Q_{it}}$ and $s_{it}^{K} = \frac{\partial Q}{\partial K}\frac{K_{it}}{Q_{it}}$, and rearranging terms, provides the equation for estimating TFP—the growth in output not explained by the growth in inputs—known as the Solow residual,

$$\frac{\dot{A}_{it}}{A_{it}} = \frac{\dot{Q}_{it}}{Q_{it}} - s_{it}^{L}\frac{\dot{L}_{it}}{L_{it}} - s_{it}^{K}\frac{\dot{K}_{it}}{K_{it}}. \tag{8}$$

Although output elasticities are not directly observable, under the assumptions of the model, capital and labor inputs can be paid the value of their marginal products. Relative prices can be substituted for the output elasticities, $s_{it}^{L} = \frac{w_{it}}{p_{it}}$ and $s_{it}^{K} = \frac{r_{it}}{p_{it}}$, where w_{it} is the price of labor, r_{it} is the price of capital, and p_{it} is the price of output. These price ratios identify the marginal rates of substitution for inputs. By using both data on prices and quantities, movements along the production function can be separated from shifts in the production function—where shifts identify TFP (Jorgensen and Griliches 1967). Importantly, the Solow residual is a true index number as it can be computed directly from prices and quantities (Hulten 2001). However, because the above equation is for continuous time, an example will be provided of the discrete-time approach (Törnqvist index), which will be discussed next.

Limitations of the Solow (1957) model include its underlying assumptions of constant returns to scale, marginal cost pricing (factors are paid their marginal products), and the nature of the technical change (by which A_{it} is assumed to affect the marginal productivity of all inputs equally) (Hulten 2001). If firms do not exhibit constant returns to scale, if imperfect competition leads to prices greater than marginal costs for inputs, or if the production function does not shift (improve productivity) by the same amount for all combinations of labor and capital, then the measure of TFP growth will be biased (Hulten 2001).

To address these biases, Jorgensen and Griliches (1967) introduced several measurement innovations to the Solow framework, including a discrete-time approximation to the Divisia index[1] approach used by Solow (1957), which was derived from the Törnqvist index. With the

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[1] A Divisia index is an approach to price and quantity measurement that assumes prices and quantities are functions of continuous time. Because prices and quantities are generally not observed continuously, many discrete time approximations have been developed. See Diewert (1988) for further reading.

Törnqvist index, the continuous time shares of labor and capital, s^L_{it} and s^K_{it}, are replaced by the average of between-period shares of labor and capital, $\frac{s^L_{it}+s^L_{it-1}}{2}$ and $\frac{s^K_{it}+s^K_{it-1}}{2}$, and continuous time growth rates are replaced with differences in natural logarithms of respective variables,

$$lnA_{it} - lnA_{it-1} = ln\frac{Q_{it}}{Q_{it-1}} - \left(\frac{s^L_{it}+s^L_{it-1}}{2}\right) ln\frac{L_{it}}{L_{it-1}} - \left(\frac{s^K_{it}+s^K_{it-1}}{2}\right) ln\frac{K_{it}}{K_{it-1}}. \qquad (9)$$

Which requires information on output, Q, inputs labor, L, and capital, K, and the relative shares of wages or capital rents included in output prices, s^L or s^K, for firm i at times t and $t-1$ (Jorgensen and Griliches 1967; Van Biesebroeck 2007). Importantly, information on the right-hand side of the equation is observable and can be used to calculate TFP growth.

To provide a brief numerical example, suppose we have a firm that produces two outputs *A* and *B* from two inputs, *X* and *Y*. The prices and quantities of outputs and costs and quantities of inputs are given in Table 2.

Table 2. Törnqvist Index Example – Output and Input Prices and Quantities

	Prices and Quantities of Outputs				**Costs and Quantities of Inputs**			
Time	P_A	Q_A	P_B	Q_B	C_X	Q_X	C_Y	Q_Y
$t-1$	3	5	3	5	2	7	3	7
t	3	6	4	6	4	6	4	11

From price and quantity or cost and quantity data in Table 2, one can compute revenue or cost shares for outputs and inputs, as shown in Table 3.

Table 3. Törnqvist Index Example – Revenue and Cost Share Calculation

	(1) Revenue from A	**(2) Revenue from B**	**(3) Total Revenue**	**(4) s^A**	**(5) s^B**	**(6) Cost of X**	**(7) Cost of Y**	**(8) Total Cost**	**(9) s^X**	**(10) s^Y**
Time	$P_A \times Q_A$	$P_B \times Q_B$	$(1) + (2)$	$(1) \div (3)$	$(2) \div (3)$	$C_X \times Q_X$	$C_Y \times Q_Y$	$(6) + (7)$	$(6) \div (8)$	$(7) \div (8)$
$t-1$	15	15	30	0.5	0.5	14	21	35	0.4	0.6
t	18	24	42	0.429	0.571	24	44	68	0.353	0.647

From this data we can compute the Törnqvist index, which shows a 4% decrease in productivity, as shown in column (7) of Table 4.[1]

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[1] Note that Törnqvist index numbers are exponentiated in Table 4.

Table 4. Törnqvist Index Example – Calculation of Output, Input, and Productivity Indexes

	(1)	**(2)**	**(3)** **Törnqvist Output Index**	**(4)**	**(5)**	**(6)** **Törnqvist Input Index**	**(7)** **Productivity Index**
Time	$\frac{s_t^A + s_{t-1}^A}{2} \ln \frac{Q_{A,t}}{Q_{A,t-1}}$	$\frac{s_t^B + s_{t-1}^B}{2} \ln \frac{Q_{B,t}}{Q_{B,t-1}}$	$e^{(1)+(2)}$	$\frac{s_t^X + s_{t-1}^X}{2} \ln \frac{Q_{X,t}}{Q_{X,t-1}}$	$\frac{s_t^Y + s_{t-1}^Y}{2} \ln \frac{Q_{Y,t}}{Q_{Y,t-1}}$	$e^{(4)+(5)}$	$(3) \div (6)$[1]
$t-1$	0 (The base year is normalized so that $\frac{Q_{A,t}}{Q_{A,t-1}}$ = 1)	0	1	0	0	1	1
t	0.085	0.098	1.2	-0.058	0.281	1.25	0.96

The Törnqvist index is frequently used in TFP analysis due to important findings from Diewert (1976), Caves et al. (1982a), and Caves et al. (1982b) who showed that the Törnqvist index could be used for multilateral productivity comparisons, meaning the index could be used to assess both levels and growth of TFP among firms (as opposed to just growth of TFP as allowed by bilateral index approaches) by comparing firm i to a hypothetical firm derived from average log output ($\overline{lnQ_t}$), labor ($\overline{lnL_t}$) , and capital ($\overline{lnK_t}$) shares from a representative sample of firms[2]

$$\overline{lnA_t} = \left(lnQ_{it} - \overline{lnQ_t}\right) - \widetilde{s^L}_{it}\left(lnL_{it} - \overline{lnL_t}\right) - \widetilde{s^K}_{it}\left(lnK_{it} - \overline{lnK_t}\right), \tag{10}$$

where $\widetilde{s^L}_{it} = \frac{s_{it}^L + s_t^L}{2}$ and $\widetilde{s^K}_{it} = \frac{s_{it}^K + s_t^K}{2}$. This allows comparisons that are bilateral and transitive for multilateral productivity studies (Caves et al. 1982b; Van Biesebroeck 2007).

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[1] Equivalent to $e^{[(1)+(2)]-[(4)+(5)]}$.

[2] Diewert (1976) showed that the Törnqvist index approximation derived by Jorgenson and Griliches (1967) is an exact index number if the production function has the translog form (Christensen et al. 1973), meaning the Törnqvist index is an exact number rather than an approximation.
Short for transcendental logarithmic production function, the translog production functions represent a class of flexible production function forms that do not have rigid assumptions, such as perfect substitution for product factors or perfect competition in factor markets. A generalized form of the translog production function for *n* inputs is $lnY = lnA + \sum_{i=1}^{n} \alpha_i \, lnX_i + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} \beta_{ij} lnXlnX_j$, where X represents inputs and Y represents output. See Pavelescu (2011) for further reading. Further, because the translog production function is a reasonable second-order approximation to other production functions, it is a sensible choice for estimating TFP and is widely used in the performance-based regulation literature.
Caves et al. (1982a) then showed that the Törnqvist index was even more broadly valid as it equals the geometric mean of two Malmquist indexes with translog production structures, allowing for the use of either firm's production technology so long as the technology was characterized by the translog (output) distance function (Van Biesebroeck 2007). The distance function is a deflation function pioneered by Malmquist, who compared the input of a firm at two different points in time by determining the maximum factor by which the input in one period could be deflated while still allowing the firm to produce the same amount of output in the other period (for the Malmquist input index). There is also a corresponding Malmquist output index. Caves et al. (1982) extended this idea to a Malmquist productivity index. Although the Malmquist index requires knowledge of the functional form for the structure of production, Caves et al. (1982a) showed that with a translog production function, the average of two firms' Malmquist indexes could be computed with only information on prices and quantities, i.e., without knowing the exact translog production function parameters. See Caves et al. (1982a) for further reading.

The Törnqvist index also closely approximates the Fisher ideal index, which has desirable axiomatic properties. However, the Fisher ideal index is only suitable for bilateral comparisons,[1] allowing for comparisons of rates of change of productivity between firms, but not comparisons of absolute levels of productivity between firms (Lawrence and Diewert 2004). Instead, to compare both levels and growth rates of productivity, practitioners use the Törnqvist index. Because the Törnqvist index is independent of which utility or year is used as the base observation, it is transitive, essentially comparing each observation to a hypothetical average firm. In practice, practitioners in ratemaking cases favor the Fisher index or the Törnqvist index, which are both superlative indexes.[2] A more detailed overview of both indexes is provided in Appendix A. See Lawrence and Diewert (2004) for further reading.

The main benefits of the index number approach are that TFP can be calculated rather than estimated, that many outputs and inputs can be incorporated, and that production technology can vary among firms. The main limitations of the approach are the underlying assumptions of firm behavior and market structure, which include an assumption of perfect competition in output and input markets, that firms have optimizing behavior, and that data used to calculate the indexes do not suffer from measurement error (Van Biesebroeck 2007). While constant returns to scale is also an underlying assumption, an additional factor for increasing or decreasing returns to scale can be included to address the scale effect (Van Biesebroeck 2007). Further, Diewert and Fox (2010) provide a correction for estimating TFP for monopolistic firms, which requires estimating marginal cost prices from econometric or accounting studies rather than using observed factor prices when there are increasing returns to scale.

3.2 Parametric Methods: Ordinary Least Squares and Other Econometric Methods

When measurement error is a concern, parametric methods are the preferred approach to TFP estimation (Van Biesebroeck 2007). Parametric methods require the estimation of a cost or production function, with the estimated function then used to identify changes in productivity or efficiency. Two prominent econometric methods for this approach are (1) to estimate the parameters of the production function, or (2) to assume that firms exhibit profit-maximizing or cost-minimizing behavior, deriving cost or demand functions (Abbott 2005). As an example, Van Biesebroeck (2007) shows that by taking the natural logarithm of a Cobb-Douglas production function, an econometrician can estimate

$$q_{it} = \alpha_0 + \alpha_l l_{it} + \alpha_k k_{it} + \omega_{it} + \epsilon_{it}, \qquad (11)$$

where q is the output of firm i at time t, l is labor, and k is capital. The term ω represents unobserved productivity differences between firms and is unobservable to the econometrician. If unobserved productivity differences are correlated with input choices (and they likely are), it will introduce simultaneity bias into the estimating equation. For this reason, estimating firm-level production functions with ordinary least squares (OLS) is generally advised against (Van Biesebroeck 2007 and Van Beveren 2008). Econometric methods such as the generalized method of moments (GMM), semi-parametric estimators, and stochastic frontier methods have been proposed in the literature to address this issue, with methods differing in the way that they

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[1] See Diewert and Nakamura (2003) for further reading.

[2] Both the Fisher index and the Törnqvist index are index numbers that are exact for flexible aggregator functions (including production functions), and for this reason, Diewert (1976) deemed these index numbers "superlative" (Caves et al. 1982a). See Caves et al. (1982a) for further reading.

deal with the problem of simultaneity of inputs and observed productivity. See Van Biesebroeck (2007) and Van Beveren (2008) for further reading.

If potential biases are sufficiently addressed with econometric methods, using the estimated parameters for labor ($\widehat{\alpha}_l$) and capital ($\widehat{\alpha_k}$), productivity growth can be estimated as

$$lnA_{it} - \overline{lnA_t} = (q_{it} - \bar{q}_t) - \widehat{\alpha}_l(l_{it} - \bar{l}_t) - \widehat{\alpha_k}(k_{it} - \bar{k}_t). \qquad (12)$$

3.3 Output and Input Measurement: In Theory

Key challenges in TFP measurement include the measurement of output, the measurement of input—especially the concept of capital—missing or inappropriate data, and the weights used for indexes (Lipsey and Carlaw 2004). To address these challenges in measurement and aggregation, a review of common methods for measuring and aggregating outputs and inputs for the electricity sector will be discussed.

First, as a reminder, the objective in price or revenue cap regulation using an external benchmark is to allow prices or revenues to rise at the macroeconomic rate of inflation, $\dot{P}^E$, minus an offset for the X-factor, which sums the difference in TFP growth rates in the electric industry and the rest of the economy ($\dot{T} - \dot{T^E}$) and the difference in input price growth rates between the rest of the economy and the electric industry ($\dot{W^E} - \dot{W}$), as shown in equation (3) and provided below for reference. Both productivity and input price differentials must be determined, requiring measurement of both output and input prices and quantities.

$$\dot{P} = \dot{P}^E - \left[\dot{T} - \dot{T^E}\right] + \left[\dot{W^E} - \dot{W}\right] \qquad (13)$$

Lawrence and Diewert (2004) point out that the main challenge for calculating TFP in the electricity sector is specifying and measuring the quantity and value of a distributor's outputs and capital inputs.

3.4 Output Measurement

TFP studies using index number methods decompose outputs into output quantity and price indexes. Because TFP reflects the difference in growth rates between a firm's physical outputs and inputs, the choice of output measure affects estimated TFP growth. Outputs can be measured from the perspective of demand or supply. The demand approach considers output to be the amount and value of energy (throughput) provided by distributors to their consumers. Although a distributor's volume of sales may represent energy throughput and total revenue represents its value, because distributors must also provide and maintain infrastructure for delivering electricity to consumers (and this is not costless), the supply approach instead considers output as a measure of the availability of infrastructure and the condition of that infrastructure. Supply-side measures of output include reliability, quality, and quantity of electricity supplied as well as coverage and capacity of the system (Diewert 2004; Lawrence 2009).

Output is typically measured as a combination of demand- and supply-side factors,[1] although experts vary in their choice of which measures to include. Some differences are driven by whether the regulation is a price or revenue cap. With a price cap a company's revenues are directly affected by how much energy is sold, and a volumetric measure of output such as volume or peak demand is common. However, with a revenue cap or a revenue-per-customer cap, the number of customers may be more important drivers for a company's revenues,[2] as an example, Lowry and Makos (2018) advocate for an output index based solely on the growth in number of customers served, based on the importance of this variable in driving utility costs. Other practitioners recommend combining several output measures to reflect changes in output trends.

As an example, Lawrence and Diewert (2004) recommend a three variable specification comprised of energy throughput, system capacity, and customers (number of connections) to incorporate both customer- and sales-density variables for measuring output for TFP analysis for Australia. Makholm (2018) relates the choice of output variables to the importance of reflecting changes in output trends due to the changing nature of investments, as an increase in inputs may not necessarily lead to an increase in output (for example investments in advanced metering infrastructure aim to reduce electricity demand). Similar to Lawrence and Diewert (2004), Makholm (2018) notes that TFP studies tend to use a mix of output measures (number of customers, line miles, peak usage, etc.) in addition to the traditional output measure (kWh) to reflect these changing output trends.

Although reliability is an important dimension (as measured by number and duration of interruptions) as a decrease in these variables indicates a reliability improvement, their inclusion requires incorporating negative outputs and inverting those negative outputs to create in increase in the overall output index – a calculation not readily addressed by most indexing methods (Lawrence and Diewert, 2004). Instead, reliability metrics are often included in performance metrics or scorecards in the performance-based regulation plan (Lowry and Makos, 2021). Regulators options to address these dimensions are typically limited to financial penalties that can be assessed for failing to meet these metrics (Meitzen et al 2017).

Aggregating disparate outputs into total output requires the use of index number methods, which require a weight be allocated to each output. A commonly used weight is the share of revenue for each output. However, if there isn't an explicit price available for each output, the revenue share has to be inferred, usually from econometric data, where a common approach is to use an econometric cost function to derive cost elasticities (Lawrence and Diewert 2004).

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[1] This is not always the case as output measures differ by how the index will be used (price or revenue cap) as well as by expert practitioner. Makholm et al. (2010) for example, consider only the volume of sales and total revenue as a measure of output. For Lowry and Makos (2018), the decision of which outputs to include in an output index is driven by how the index will be used, for example, for a revenue cap index, determinants that affect cost are more relevant as revenue trends should track cost trends. Included outputs should measure trends in workload that drive costs, such as the growth in the number of customers, with the weight of each included determinant representative of its share in costs. Whereas, determinants that affect revenue (billing determinants, weighted by their share of revenues) are more relevant for price cap indexes (Lowry 2018).

[2] https://www2.auc.ab.ca/h002/Proceeding566/ProceedingDocuments/2012-237%20R_2239.PDF (accessed 6/7/2022).

Table 5 provides a summary of outputs used in recent TFP studies.[1]

Table 5. Summary of Output Measurement for Recent TFP Studies

Study	Price/Revenue Cap or Other	Output	Weights	Methods
"Designing Revenue Adjustment Indexes for Hawaiian Electric Companies" (Lowry, 2019) and "New X Factor Research for HECO" (Lowry et al., 2020)[2] [3]	Revenue cap	Number of customers, ratcheted maximum peak demand, mid-year generation capacity, generation volume, mid-year transmission line miles	Cost shares were computed with an econometric cost model	Törnqvist index methodology
"2018 – 2022 Performance-Based Regulation Plans for Alberta Electric and Gas Distribution Utilities (Errata to Decision 20414-D01-2016)."[4] *Note that this study primarily leveraged the Makholm and Ros (2010) study methodology described below, with modifications as noted See written evidence of Dr. Brown and Dr. Carpenter (Brown and Carpenter, 2016)[5]	Rate (price) cap for electric distribution companies; revenue-per-customer cap for gas distribution utilities	Volume (MWh) residential, commercial, industrial, and public sales)	Revenue-based weights	Törnqvist index methodology

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[1] This survey focuses on more recent TFP studies with an external benchmark (rather than a utility's own forecasted costs and productivity). For a survey of previous literature on TFP studies, see the Australian Competition and Consumer Commission/Australian Energy Regulator (ACCC/AER) Working Paper 6 /May 2012 available at https://www.accc.gov.au/system/files/Working%20paper%20no.%206%20%20-%20Benchmarking%20energy%20networks.pdf (accessed 6/9/2022).

[2] https://dms.puc.hawaii.gov/dms/DocumentViewer?pid=A1001001A20E14B04623B00782 (accessed 6/8/2022).

[3] The proposed X-factor resulting from this TFP study was not accepted by the Hawaii PUC, but methodology is provided for comparison purposes. https://dms.puc.hawaii.gov/dms/DocumentViewer?pid=A1001001A19H15A91714G00161 (accessed 6/8/2022).

[4] https://www2.auc.ab.ca/h007/Proceeding20414/ProceedingDocuments/20414-D01-2016Errata2018-2022PBRPlansfor_0712.pdf (accessed 6/6/2022). https://www.regulatorylawchambers.ca/blog/2018/12/10/decision-20414-d01-2016-re-2018-2022-pbr-plans-for-alberta-electric-and-gas-distribution-utilities (accessed 6/6/2022).

[5] https://www2.auc.ab.ca/h007/Proceeding20414/ProceedingDocuments/AppendixA-BrattleWrittenEvidence_0059.pdf (accessed 6/14/2022). https://www2.auc.ab.ca/h007/Proceeding20414/ProceedingDocuments/2016-05-27-Brattlereplyevidenceonnextgen_0397.pdf (accessed 6/14/2022).

Study	Price/Revenue Cap or Other	Output	Weights	Methods
"Determination of the Second-Generation X Factor for the AUC[1] Price Cap Plan for Alberta Electric Distribution Companies" (Meitzen, 2016)[2] *Utilized the Makholm and Ros (2010) TFP methodology but updated the sample for more recent years	Rate (price) cap for electric distribution companies; revenue-per-customer cap for gas distribution utilities	Volume (MWh) (residential, commercial, industrial, and public sales)	Revenue-based weights	Törnqvist index methodology
"Next Generation PBR[3] for Alberta Energy Distributors" (Lowry, 2016)[4]	Rate (price) cap for electric dist. companies; revenue-per-customer cap for gas dist. utilities	Number of customers (total number of retail customers served)	Construction of output index weights were not discussed	Törnqvist index methodology
"Productivity and Benchmarking Research in Support of Incentive Rate Setting in Ontario" (Kaufmann et al., 2013)[5]	Price cap	Customer numbers (other than street lighting, sentinel lighting, and unmetered scattered loads), total kWh deliveries, and system capacity peak demand	Each output's cost elasticity share is derived from an econometric cost model	Törnqvist index methodology
"Total Factor Productivity Study for Use in AUC Proceeding 566 – Rate Regulation Initiative," December 30, 2010. (Makholm and Ros, 2010)[6]	Rate (price) cap for electric distribution companies; revenue-per-customer cap for gas	Residential, commercial, industrial, and public sales volume	Revenue-based weights derived from electric sales for each of the	Törnqvist index methodology

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[1] Alberta Utilities Commission.

[2] https://www2.auc.ab.ca/h007/Proceeding20414/ProceedingDocuments/EDTINextGenerationPBRPlanSubmission_0076.pdf (p. 185, accessed 6/15/2022).

[3] Performance-based regulation.

[4] https://www2.auc.ab.ca/h007/Proceeding20414/ProceedingDocuments/CCAEvidenceofPEG_0084.pdf (accessed 6/14/2022).

[5] https://www.oeb.ca/oeb/_Documents/EB-2010-0379/EB-2010-0379_Final_PEG_Report_20131111.pdf (accessed 6/9/2022).

[6] https://www2.auc.ab.ca/h002/Proceeding566/ProceedingDocuments/1a_ID566%20N_0204.pdf (accessed 6/22/2022).

Study	Price/Revenue Cap or Other	Output	Weights	Methods
"Update, Reply and PBR Plan Review for AUC Proceeding 566 – Rate Regulation Initiative," February 22, 2012. (Makholm and Ros, 2012)[1]	distribution utilities		customer categories	
"Total Factor Productivity and Performance-Based Ratemaking for Electricity and Gas Distribution" (Makholm et al., 2010)	Other: TFP analysis for the U.S. Electric Industry (1972–2009)	Sales volume (MWh)	Revenue-based weights	Törnqvist index methodology

3.5 Input Measurement

Input indexes are used to capture growth in input quantities and growth in input prices, as both components make up the growth in company costs and are necessary for calculating the X-factor. Input indexes are typically comprised of multiple inputs, with distribution systems typically including two broad categories: operations and maintenance expenditure and capital expenditure (Lawrence 2009). In North America, operations and maintenance is often separated into labor, materials, and services (Lawrence 2009). The weights of input indexes are determined by the relative cost share of each input to the total cost of all inputs, with capital subindexes typically being allocated the heaviest weights as distribution systems are capital intensive (Lowry and Makos 2018).

3.5.1 Labor, Materials, and Services

There are a few different approaches for measuring labor or materials and services quantity and cost. Quantity can be measured directly when data permits, for example, labor quantity can be measured with the number of full-time employees, although labor input data is increasingly difficult to obtain due to contracted labor services (Lawrence 2009). Alternatively, quantity can be measured indirectly by deflating the value of relevant costs. For example, labor costs (measured by salary and wage expenses) can be deflated by relevant labor price indexes (measured by a salary and wage price index) to obtain implicit quantity measures.

3.5.2 Capital

There are a number of different approaches for measuring capital quantity and cost. Capital is a unique input as it is purchased in one time period but delivers a flow of services over time. Therefore, the purchase cost of the capital asset must be distributed somehow over the useful life of the asset (Diewert 2003). Hulten (1991) characterizes the challenge for capital measurement as follows:

> *Durability means that a capital good is productive for two or more time periods, and this, in turn, implies that a distinction must be made between the value of using or renting*

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[1] https://www2.auc.ab.ca/h002/Proceeding566/ProceedingDocuments/Second%20Rep_1425.pdf (accessed 6/22/2022).

> *capital in any year and the value of owning the capital asset. The distinction would not necessarily lead to a measurement problem if the capital services used in any given year were paid for in that year, that is, if all capital were rented. In this case, transactions in the rental market would fix the price and quantity of capital in each time period, much as data on the price and quantity of labor services are derived from labor market transactions. But, unfortunately, much capital is utilized by its owner and the transfer of capital services between owner and user results in an implicit rent typically not observed by the statistician. Market data are thus inadequate for the task of directly estimating the price and quantity of capital services, and this has led to the development of indirect procedures for inferring the quantity of capital, like the perpetual inventory method, or to the acceptance of flawed measures, like book value.*

TFP studies using index number methods decompose capital cost into consistent capital quantity and price indexes. The capital quantity index often measures the flow of services from the acquired capital assets and the capital price index measures the prices that would be earned in a competitive market for the rental of capital services (Lowry and Makos 2018)—a price that has to be inferred as most capital is owned by the distribution company.

3.5.2.1 Capital Quantity

Capital quantity can be measured directly, for example, with a measure of line length or transformer capacity. Alternatively, capital quantity can be measured indirectly with a constant price depreciated asset value (the deflated asset value method) (Lawrence and Diewert 2004; Lawrence 2009). With the deflated asset value method, the capital quantity index is constructed by deflating data on the value of assets—for example, a utility plant value is deflated using a construction cost index (Lowry 2019). Other complications include whether the asset should be valued at current (replacement) value or historical (book) value, as replacement value methods require implicit capital gains to be netted off of the cost of capital when asset prices rise; and the assumed pattern of depreciation of assets, which should be reflected in both capital price and quantity indexes (Lowry and Makos 2018).

Often, a practitioner will observe the new capital (I_t) added to the capital stock (K_t) each year, but not the total capital stock at that point in time. The total capital stock will need to be inferred from past and current additions, accounting for the possibility that older capital may be less productive (Hulten 1991). In practice, because TFP studies use constant or real dollars[1] and depreciation patterns may vary from those used at utilities for valuing capital assets, it is typical for capital to be valued based on capital additions in each year of the study rather than using the gross or net plant balances in utility accounts. However, this requires determining a benchmark year, or the opening balance, at the start of the study, which is developed by using gross or net plant balances in that year. One method for adding up capital additions (I) into capital stock (K) is the perpetual inventory method. With this method, investment (I) from all surviving capital vintages is weighted by a number (ϕ) between zero and one to allow for older capital to be less productive than newer capital, and summed to equal a total capital measure:

$$K_t = \phi_0 I_t + \phi_1 I_{t-1} + \cdots + \phi_t I_{t-T}, \qquad (14)$$

where $\phi_0 = 1$ and $t - T$ is the date of the oldest surviving vintage (Hulten 1991). Equation (14) defines the capital stock in efficiency units, which requires an estimation of efficiency weights

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[1] Capital asset value after adjusting for inflation.

(ϕ), which are rarely observed. One approach is to estimate the relative efficiency indirectly by assuming ϕ follows an observable pattern (Hulten 1991). The decision whether to use physical quantity or constant price depreciated asset values to measure annual capital inputs relates to the underlying assumption about the relative efficiency of assets (Lawrence 2009).

Underlying the constant price depreciated asset value method is an implicit assumption of geometric or straight-line depreciation, whereas physical measures assume a "one-hoss-shay" depreciation profile (Lawrence 2009)[1]. The specific model of depreciation chosen implies different measures for the flow of services from capital, which will lead to different measures of TFP growth (Diewert and Lawrence 2000).

Depreciation Profiles

In economic theory, physical asset depreciation is equal to the reduced efficiency, or decline in value, as an asset progresses in age. Three depreciation patterns are predominantly used to capture changes in relative efficiency over time: one-hoss-shay, straight-line, and geometric decay.

With the "one-hoss-shay" efficiency pattern, the efficiency of an asset (ϕ) over the service life of the asset ($t = 0, 1, 2, \dots, T$) is assumed to be fully efficient (i.e., equal to one) until the asset falls apart when the service life ends (i.e., beyond the service life asset efficiency is equal to zero). This efficiency pattern is completely determined by the service life of the asset (Hulten and Wykoff 1996):

$$\phi_0 = \phi_1 = \dots = \phi_{T-1} = 1, \phi_{T+t} = 0. \tag{15}$$

With the straight-line efficiency pattern, is it assumed that efficiency declines linearly until the asset is retired, and again is determined by the service life of the asset, although efficiency decays in equal increments ($1/T$) each year (Hulten and Wykoff 1996).

$$\phi_0 = 1, \phi_1 = 1 - \left(\frac{1}{T}\right),\ \ \phi_2 = 1 - \left(\frac{2}{T}\right), \dots, \phi_{T-1} = 1 - \left(\frac{T-1}{T}\right), \phi_{T+t} = 0. \tag{16}$$

With the geometric decay efficiency pattern, it is assumed that the productive capacity of the asset decays at a constant rate, $\delta = \frac{\phi_{t-1} - \phi_t}{\phi_{t-1}}$, giving an efficiency sequence,

$$\phi_0 = 1, \phi_1 = (1 - \delta), \phi_2 = (1 - \delta)^2, \dots, \phi_t = (1 - \delta)^t\ , \tag{17}$$

which is characterized by the decay rate δ rather than the service life of the asset (Hulten and Wykoff 1996).

Although the efficiency parameters in the above equations represent the efficiency of an x-year-old asset relative to a new asset (where a new asset has efficiency $\phi_0 = 1$); in practice, they are often estimated from the rental prices of capital. These efficiency patterns point to the appropriate method that should be used for economic depreciation. If an asset's price declines linearly with age, depreciation should be the straight-line form. If the asset's price declines more slowly than the straight-line pattern as the asset ages; this indicates a one-hoss-shay pattern where the asset retains its full productive capacity until retirement (or straight-line depreciation with a zero rate of discount). If an asset's price declines at a constant rate with age; this implies geometric decay is the most appropriate assumption.

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[1] One hoss shay is a method of depreciation where the annual value is equal until its value is zero.

3.5.2.2 Capital Cost

The annual cost of using capital inputs can be measured directly by applying a constant percentage reflecting depreciation, the opportunity cost of capital, and the rate of capital gains to the value of assets. Or the annual cost of using capital inputs can be measured indirectly as the residual from the equation: revenue minus operating costs. The direct method of measuring capital cost better reflects producer theory from economics as it is an ex-ante (rather than ex-post) measure of capital cost but can be problematic if sampled firms earn a wide range of rates of return (Lawrence and Diewert 2004).

The direct approach to measuring capital costs requires the application of a “user cost” (which reflects depreciation, the opportunity cost of capital, and capital gains) to the value of the assets (Lawrence 2009). The basic formula for user cost, assuming an asset is purchased and used for one period, is

$$U_t = P_t - (1+r)^{-1} P_{t+1} \tag{18}$$

where the user cost of an asset that is t years old (U_t) is equal to its purchase price (P_t) minus the discounted end of period price one year in the future, $(1+r)^{-1}P_{t+1}$, where the real interest rate is r.

Geometric Decay

With geometric decay as the depreciation assumption, the net capital stock model[1] is appropriate for aggregating vintages of capital stock,

$$K = I_0 + (I - \delta)I_1 + (1-\delta)^2 I_2 + \cdots + (1-\delta)^t I_t \tag{19}$$

where K is the capital stock, aggregated over all vintages, I_0 is the new investment in the asset in the current period, and I_t is the vintage investment that occurred t periods ago (for $t = 1, 2, \ldots$,t).[2] Geometric decay is a common depreciation assumption in many X-factor studies in the energy industry as geometric decay has been shown to accurately characterize depreciation in many industries (Hulten and Wykoff 1980; Lowry and Makos 2018).

With the geometric decay, also called the declining balance assumption, the rental price for a new asset is equal to

$$U_0 = (1+r)^{-1}(r+\delta)P_0 \tag{20}$$

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[1] This model starts with Jorgensen (1963) but has been used extensively in the literature. See Diewert and Lawrence (2000) for further reading.

[2] With this model, it may not be necessary to use index number methods to aggregate over capital vintages (i.e., linear aggregation is appropriate) if we assume that the capital services from each vintage of a homogeneous type of capital are perfectly substitutable; see Diewert and Lawrence (2000) for further reading.

where U_0 is the rental price (user cost) and P_0 is the asset price at time zero (that is, when the asset is new). The real interest rate is r, and the constant rate of depreciation is δ. The rental price[1] for a t year old asset is then,

$$U_t = (1-\delta)^t U_0. \tag{21}$$

The value of the capital stock is equivalent to

$$U_0 I_0 + U_1 I_1 + \cdots = U_0[I_0 + (1-\delta)I_1 + (1-\delta)^2 I_2 + \cdots + (1-\delta)^t I_t]. \tag{22}$$

This model is equivalent to a price term U_0 times the aggregate capital stock shown in equation (19).

In practice, with geometric decay as the depreciation assumption, capital assets are valued in replacement dollars.[2] In addition, capital prices used are consistent with the geometric decay assumption shown in equation (20).

As an example, in “Designing Revenue Adjustment Indexes for Hawaiian Electric Companies” and “New X Factor Research for HECO,”[3] [4] Lowry (2019) constructs the capital quantity index with the assumption of geometric decay for asset depreciation.

The capital quantity index was constructed as follows. In the base year, plant quantity was estimated by deflating the value of the plant reported in FERC Form 1 (book value) by applicable construction cost indexes (Handy-Whitman Index of Cost Trends of Electric Utility Construction for Total Plant – All Steam Generation). In subsequent years, the following model (assuming depreciation followed a geometric decay pattern) was used:

$$XK_t = (1-d) * XK_{t-1} + \frac{VI_t}{WKA_t}, \tag{23}$$

where XK is capital quantity, d is the economic depreciation rate, VI is gross additions to plant and WKA is the construction cost index; t indexes time.[5]

The corresponding capital service price also reflects the geometric decay assumption:

$$WKS_{j,t} = d * WKA_{j,t} + WKA_{j,t-1}\left[r_t - \frac{WKA_{j,t} - WKA_{j,t-1}}{WKA_{j,t-1}}\right], \tag{24}$$

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[1] The formula implies that the asset rental price varies in fixed proportion over time and allows the capital stock to be aggregated without index number theory; see Diewert and Lawrence (2009) for further reading.

[2] As discussed by Pacific Economic Group (PEG) in their TFP analysis for the Hawaiian Electric https://dms.puc.hawaii.gov/dms/DocumentViewer?pid=A1001001A19H15A91714G00161 (PDF p. 95).

[3] https://dms.puc.hawaii.gov/dms/DocumentViewer?pid=A1001001A20E14B04623B00782 (accessed 6/8/2022).

[4] Note this X-factor was not accepted by the Hawaii PUC, but methodology is provided for comparison purposes https://dms.puc.hawaii.gov/dms/DocumentViewer?pid=A1001001A19H15A91714G00161 (accessed 6/8/2022).

[5] Note that PEG revised their methodology to use straight-line depreciation and prices in a later filing.

where WKS is the capital price, d is the economic depreciation rate, WKA is the construction cost index, and r is the weighted average nominal price of capital, the second term was smoothed to reduce capital cost volatility; t indexes time.

One-Hoss-Shay

With "one-hoss-shay" as the depreciation assumption, the gross capital stock model is appropriate for aggregating vintages of capital stock,

$$K = I_0 + I_1 + \ldots + I_{N-1}, \tag{25}$$

where K is the capital stock, aggregated over the current period investment I_0 and all other investments in $N-1$ prior periods.[1] The one-hoss-shay method of depreciation is the most commonly used assumption for electric utility X-factor studies due to the method reflecting that the constant service flow from assets is arguably more appropriate than a gradual decline (Lowry and Makos 2018). The capital quantity index is typically the inflation-adjusted gross plant value, which rises with additions and falls with retirements (Lowry and Makos 2018). Further, some practitioners invoke the one-hoss-shay assumption when using physical asset measures for capital quantity (Lawrence and Diewert 2004; Lowry and Makos 2018)

With the "one-hoss-shay" depreciation assumption, the rental price for a new asset is

$$U_0 = P_0 r(1+r)^{-1}[1-(1+r)^{-N}]^{-1}, \tag{26}$$

where U_0 is the rental price[2] (user cost) and P_0 is the asset price at time zero (when the asset is new). The real interest rate is r and the useful life of the asset is N. Capital value is then equivalent to the rental price, U_0, times the aggregate capital stock shown in equation (25). In practice, assets are valued at replacement cost and cost is computed net of capital gains.[3] Capital prices are also consistent with the one-hoss-shay assumption shown in equation (26). Note that the capital price is a function of prices of new assets only, an assumption which can create large difference in capital valuation (Lowry and Makos 2018).

As an example, Makholm et al. (2010) construct capital quantity and price indexes with one-hoss-shay depreciation assumptions in measuring TFP trends for electric companies in the United States. The book value of the plant and the Handy-Whitman Index (*HW*) is used to compute capital quantity for the benchmark year ($K_{benchmark}$).

$$K_{benchmark} = \frac{book\ value\ of\ plant_{benchmark}}{\sum_{i=1}^{20} i \left[\frac{i}{\sum_{i=1}^{20} i}\right] HW_{1994+i}} \tag{27}$$

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[1] With this model, it is again unnecessary to use index number methods to aggregate over capital vintages if we assume perfect substitution for different vintages of each homogeneous type of capital; see Diewert and Lawrence (2000) for further reading.

[2] Which again implies that the asset rental price varies in fixed proportion over time and allows the capital stock to be aggregated without index number theory.

[3] As discussed by PEG in their TFP study for Hawaiian Electric. https://dms.puc.hawaii.gov/dms/DocumentViewer?pid=A1001001A19H15A91714G00161 (PDF p. 96, accessed 5/26/2022).

After the benchmark year, capital is added according to the following formula:

$$K_t = K_{t-1} + \frac{gross\ additions\ to\ plant}{HW_t} - \frac{retirements_t}{HW_{t-s}} \tag{28}$$

Where s is the useful life of the asset. The “one-hoss-shay” depreciation pattern is used for depreciation of capital.

Capital prices (P) are based on the acquisition price of new capital and the present value of all of its future services:

$$P_{k,t} = \left(\frac{1-k-uz}{1-u}\right)(r-i)\left[1-\left(\frac{1+i}{1+r}\right)^s\right]^{-1} HW_{t-1} \tag{29}$$

Where: *k* = investment tax credit rate, *u* = corporate profits tax rate, *z* = present value of depreciation deduction on new investment, *r* = cost of capital, *i* = expected inflation rate over the asset’s lifetime, and HW_{t-1} = Handy-Whitman’s asset price in the prior year.

Straight-Line Depreciation

With the straight-line depreciation assumption, assuming the real interest rate r does not vary over time,[1]

$$K = \left(\frac{1}{N}\right)[NI_0 + (N-1)I_1 + (N-2)I_2 + \cdots + (1)I_{t-N}]\,. \tag{30}$$

The assumption of straight-line depreciation and historic valuation of an asset is most similar to the cost-of-service approach, although the model, as shown in equation (30), is more complicated than either the geometric decay or one-hoss-shay models.

With the straight-line depreciation assumption, the rental price for a new asset is

$$\begin{gathered} U_t = (1+r)^{-1}[r + N^{-1} - tN^{-1}r]P_0 \quad for\ \ t = 0,1,\ldots,N-1\ and \\ U_t = 0\ \text{ for }\ t = N,\ N+1, \end{gathered} \tag{31}$$

noting that the price for an asset at $t\ =\ 0$ is $(1+r)^{-1}\left[r+\left(\frac{1}{N}\right)\right]P_0$ and the price for an asset at time t is $t\ =\ N-1$ is $\left[\frac{1}{N}\right]P_0$.[2]

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[1] However, in the real world, interest rates do vary over time, requiring more complicated aggregation methods—that is, index number theory and superlative index number formula to aggregate over the capital stock components $(I_0, I_1, \ldots, I_{N-1})$ (Diewert and Lawrence 2000).

[2] The straight-line depreciation assumption for capital price shows that the price of the asset will not vary in strict proportion over time unless the real interest rate is constant over time (Diewert and Lawrence 2000). In reality, user costs, as shown in equation (31), can be used as the corresponding prices for the capital stock components in equation (30) used in index number theory.

As an example, Lowry (2016)[1] measured the robustness of capital cost and quantity for a TFP study for the Consumer Choice Advocate (CCA) in Alberta, Canada, using the straight-line depreciation method.

For this study, the quantity of capital (in real dollars) was:

$$xk_t = \sum_{s=1}^{N-1} \frac{N-s}{N} a_{t-s} \tag{32}$$

Where xk_t is the quantity of plant available for use in year *t* and N is the service life of the utility plant; a_{t-s} is the quantity of plant additions in year $t - s \ = \frac{VK_{t-s}^{add}}{WKA_{t-s}}$, where the term VK_{t-s}^{add} is the gross value of the plant installed in year $t - s$ and WKA_{t-s} is the price of capital assets in year $t - s$.

The price of capital was:

$$WKS_t = r_t \left(\sum_{s=0}^{N-1} \frac{xk_{t-1}^{t-s}}{xk_t} WKA_{t-s} \right) + \sum_{s=0}^{N-1} \left(\frac{xk_t^{t-s}}{xk_t} WKA_{t-s} \right) \left(\frac{1}{N-s} \right) \tag{33}$$

The indirect approach to measuring capital cost is relatively simpler and instead allocates a residual (ex-post) cost to capital, which is based on the difference between revenue and operating costs.

Measuring capital quantity with the deflated asset value method can provide a superior estimate of the capital input as it reflects the quality of capital and can include other capital items besides lines and transformers (Lawrence and Diewert 2004). However, the deflated asset value method is more appropriate for mature systems with consistent asset valuation over time and across organizations. A second concern is that the deflated asset value method usually incorporates a declining balance (geometric) approach to measure depreciation, which can be problematic if electricity assets tend to be long lived and provide a relatively constant flow of services over their lifetimes—a depreciation profile better reflected by a one-hoss-shay depreciation assumption (Lawrence and Diewert 2004).

Although the preceding discussion shows how different depreciation assumptions affect both user costs (rental prices) and capital quantities as well as potential aggregation methods, in reality other factors such as taxes and incentives will affect capital costs, requiring user costs that take these factors into account (see Christensen and Jorgensen (1969) for further reading).

3.5.3 Input Index Quantity and Price Measurement

Table A.1 in Appendix A provides an overview of common assumptions used in measuring input quantities and prices, as well as index methodologies and weights. Although the capital quantity

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[1] https://www2.auc.ab.ca/h007/Proceeding20414/ProceedingDocuments/CCAEvidenceofPEG_0084.pdf (accessed 6/14/2022).

and price models use different notation and include some additional terms, the basic structure of the models is as documented in the preceding discussion.

Beyond indexing methodology, output and input measurement, other important dimensions of a TFP study include selection of a peer group and the length of the study. These variables will be discussed in the next section.

4.0 Lessons Learned

Multi-year rate plans exist in several countries, including the United States, but the approach to conducting a TFP study varies. In the United States, several utilities operate under multi-year rate plans, but annual revenue adjustments tend to be based on utility-specific multi-year cost forecasts.[1] Other countries—including the United Kingdom and Australia—also use utility-specific multi-year cost forecasts but approve of the forecasts through economic benchmarking studies.[2] Most relevant for the approach of developing the X-factor by comparing industry TFP to an external benchmark is the TFP approach utilized in Alberta, Canada,[3] and recently in Hawaii. As Hawaii ultimately decided on an X-factor of zero percent that was outside the range of all submitted TFP studies (but in line with historic practice), this lessons learned review will focus on the Canadian experience and relevant findings within the broader TFP literature.

The Alberta Utilities Commission (AUC) began a rate regulation initiative to implement performance-based regulation in Alberta in 2010 for electric and natural gas distribution companies in the province.[4] In its 2017 decision on the next generation of performance-based regulation plans for the 2018 to 2022 period, the AUC had the unique position of evaluating

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[1] In a report for the Hawaiian Electric Companies in 2019, the Brattle Group identified seven utilities in the United States with multi-year rate plans (PG&E, Georgia Power, Public Service Company of New Hampshire, Consolidated Edison, Northern States Power, Puget Sound Energy, and Eversource), but only Eversource's annual revenue adjustment was indexed by an external benchmark; other utilities use utility-specific methods. See https://dms.puc.hawaii.gov/dms/DocumentViewer?pid=A1001001A19H15A91714G00161 (p.164, accessed 6/8/2022) and https://emp.lbl.gov/sites/default/files/reports/lbnl-1004130_0.pdf (accessed, 6/10/2022) for further reading.

[2] In the United Kingdom, the performance-based regulation framework is based on the concept that Revenues = Incentives + Innovations + Output (RIIO). The Office of Gas and Electricity Markets (OFGEM) employs econometric cost benchmarking techniques to determine overall total expenditure (TOTEX, which is CAPEX + OPEX) allowance.[2] With this approach, the X-factor is primarily determined by the forecasted cost data specific to utility using frontier methods as discussed in Appendix A. Similarly, the Australian Energy Regulator (AER) uses economic benchmarking to measure how productively distribution network service providers (DSNPs) deliver electricity distribution services over time compared with their peers. Economic benchmarking is used to determine whether total operating and capital expenditure forecasts for each business reflect the efficient cost of providing electricity. The AER uses three types of benchmarking techniques: productivity index numbers, econometric operating expenditure cost function models, and partial performance indicators. Although the AER does use multilateral TFP analyses, these index number methods are primarily used to compare productivity differentials across DNSPs. For further reading, see: https://www.aer.gov.au/system/files/AER%20Better%20Regulation%20factsheet%20-%20expenditure%20forecast%20assessment%20guideline%20-%20November%202013.pdf; https://www.aer.gov.au/system/files/Economic%20Insights%20-%20Response%20to%20consultants%20%20reports%20on%20AER%20economic%20benchmarking%20-%20April%202015_4.PDF; https://www.aer.gov.au/system/files/Economic%20Insights%20%E2%80%93%20%20Economic%20benchmarking%20assessment%20of%20operating%20expenditure%20for%20NSW%20and%20ACT%20Electricity%20DNSPs%20%E2%80%93%2017%20November%202014_1.PDF (accessed 6/10/2022).

[3] Multi-year rate plans exist in British Columbia, Alberta, and Ontario. In Alberta, rates are adjusted according to an external benchmark with a utility-specific adjustment for capital (K-factor), whereas British Columbia and Ontario have an external benchmark for O&M and a utility-specific capital tracker.

[4] See Decision 2012-237, available at https://www2.auc.ab.ca/h002/Proceeding566/ProceedingDocuments/2012-237%20R_2239.PDF (accessed 6/10/2022).

three different TFP studies, two of which had fundamental differences, allowing the AUC to evaluate the impact of a variety of assumptions.[1]

Some key takeaways related to transparency of the studies, and the sensitivity of calculations to assumptions include that study methodologies and assumptions should be transparent enough that the study could be reproduced, and sensitivity of assumptions should be documented:

> *...studies must provide information describing all aspects of the study, with considerable detail – including easily reproducible supporting calculations – on the effects, both separately and jointly, of changing each of the assumptions used, where the set of assumptions is widely defined, and includes assumptions with respect to data source selection (AUC 2017).*

On output measures, two of the TFP studies included a volumetric output measure (MWh of electricity), and one study used the number of customers. Although the Commission determined that either measure of output was valid, it recommended future TFP growth studies use a combination of output measures and examine the sensitivity of TFP growth results to different output measures:

> *…the Commission believes that a useful way to proceed in future TFP growth studies might be to use some combination of the output measures, and, as a starting point, to examine the sensitivity of the TFP growth results to different combinations of output measures. Based on analysis presented in this proceeding, however, changing the output measure leads to moderate variability in output growth, and hence, in TFP growth (AUC 2017).*

On capital, the Commission determined that the capital tracker mechanism it adopted for the 2012–2017 performance-based regulation plan had the unintended effect of placing a considerable amount of capital outside of the incentive-enhancing I-X mechanism. Instead, capital trackers were administered in a manner similar to cost-of-service regulation. To address this concern, for the 2018–2022 performance-based regulation plan, the Commission adopted a K-bar capital mechanism,[2] which it expected would provide necessary incremental funding for distribution utilities while significantly enhancing incentives to plan, design, and construct capital assets.

On the length of the study, the Commission retained its view from Decision 2012-237 that TFP studies should use the longest available time period to best reflect long-term TFP growth for

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[1] See the 2018–2022 Performance-Based Regulation Plans for Alberta Electric and Gas Distribution Utilities
(Errata to Decision 20414-D01-2016)
https://www2.auc.ab.ca/h007/Proceeding20414/ProceedingDocuments/20414-D01-2016Errata2018-2022PBRPlansfor_0712.pdf (accessed 6/6/2022).

[2] The capital tracker was administered through a cost-of-service process, where a capital funding shortfall was identified, and additional funding was provided as needed. After capital projects were completed, additional funding was provided with a true-up proceeding. With the K-bar approach, more capital expenditures were brought into the PBR framework with a determination of a base amount of additional funding required for capital in the initial PBR term that would be increased annually by index, the K-bar mechanism was agreed for providing additional capital funding. See https://www.regulatorylawchambers.ca/blog/2018/12/9/rebasing-for-the-2018-2022-pbr-plans-for-alberta-electric-and-gas-distribution-utilities-first-compliance-proceeding-decision-22394-d01-2018 (accessed 6/10/2022).

determining the X-factor (which relies on the underlying theory that the X-factor should mimic long-run growth in a competitive market). This reasoning was based on evidence provided by NERA Economic Consulting (NERA) (see Makholm and Ros, 2010) that "…TFP growth analysis should span a sufficient number of years to mitigate the effects of business cycles or other idiosyncratic swings associated with annual changes in the use of inputs and outputs, for example major capital replacements" (AUC 2012). In addition, departures from using the longest time period available should be based on justification that a structural break has occurred, evidencing that long-term growth trends are not stable. For examining the potential of a structural break, NERA recommended a two-step process: "…first, it is necessary to postulate a theory about why a structural break could have occurred, and second, it is necessary to perform a number of statistical tests to see if the postulated hypothesis is supported by the data"[1] (AUC 2012). Ultimately, the commission recommended the length of the study period should "… smooth out the effect of cost and output volatility and capture the TFP growth trend that is most likely to be pertinent during the PBR plan period" (AUC 2012). A sample period of at least 10 years was agreed upon by the experts for the purpose of determining the long-term industry TFP.

On sample selection, the Commission determined that a TFP study sample must be large enough to determine robust estimates. They found that it is acceptable to base the TFP study on either all companies in an industry for which good data are available or to select a sub-sample *if* the sub-sample is large enough to provide a reliable estimate of productivity growth. The Commission also highlighted feedback from NERA that when examining productivity growth rates (instead of productivity levels), a TFP study examines how the ratio of inputs to outputs changes over time, and as such, the unique cost features of any particular company cancel out in the process, noting that "…the standard approach in North American PBR regulatory jurisdictions is to compare each company to the industry performance and not to specific peer groups." This led the Commission to conclude:

> *…when it comes to the sample size and the use of U.S. data in TFP studies, the relevant question to ask is not whether the companies in the sample are similar to the Alberta utilities, but: (i) whether the sample in the TFP study is reflective of the productivity trend in the U.S. power distribution industry, and (ii) whether the U.S. industry TFP trend represents a reasonable productivity trend estimate for the Alberta companies (AUC 2012).*

Considering the varied assumptions employed in TFP studies, many of which were deemed reasonable, on determining the overall X-factor, the Commission decided that TFP growth cannot be determined as a single number, but rather, as a number that falls within a reasonable range of values:

> *The Commission has determined an X factor, using its judgement and expertise in weighing the evidence and in taking into account the multitude of considerations set out above, in particular evidence demonstrating that the TFP growth value cannot with certainty be identified as a single number, but rather, in view of the variability resulting from the assumptions employed, must be considered as falling within a reasonable range of values…(AUC 2017).*

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[1] https://www2.auc.ab.ca/h002/Proceeding566/ProceedingDocuments/2012-237%20R_2239.PDF (accessed 6/7/2022).

Lessons learned from Alberta on study transparency, sensitivity of TFP growth and the X-factor to underlying assumptions, including output and capital measurement, the length of the study, and sample selection, fall largely in line with broader lessons learned from the literature.

The incentives faced by a firm depend on the design of the performance-based regulation program. Bell (2002) highlights two key elements for program design: 1) de-linking a utility's own costs with its own allowed prices or revenues; and 2) linking the utility's own allowed prices or revenues with the costs of other, comparable utilities.

While a price or revenue cap decouples allowed prices or revenue from a company's costs, the sample selection of a TFP study determines how the utility's allowed prices or revenues are linked to the costs of comparable utilities. Two questions regarding the appropriateness of firms selected for comparison are: (1) as utilities are heterogeneous, does the peer group selected facilitate a meaningful comparison, and (2) how are exogenous differences between utilities accounted for? Weisman (2018) highlights that in North America, the X-factor is commonly determined based on the productivity growth of a representative sample of firms that constitutes the electric industry. When the TFP metric is productivity growth, heterogeneity across firms largely vanishes, and it is advisable to use the largest possible sample of firms. If there is reason to believe that heterogeneity persists, a sample can be restricted to more comparable firms; however, care must be taken to account for exogenous factors that drive productivity differences across firms (Weisman 2018).

With regard to the length of the study, the TFP study and the X-factor can reflect long- or short-run trends. Short-run trends can be more volatile due to input price or demand fluctuations; if the X-factor is calibrated to reflect the industry's long-run TFP trend, it can smooth these effects. However, in times of input price volatility, basing the X-factor on long-run trends can cause financial distress for utilities (Lowry and Getachew 2009). Makholm (2018) advocates for longest time period available to uncover the long-run productivity trend of the industry rather than the trend of an underlying business cycle. In assessing TFP trends from 1971–2009 and again from 2010–2017, Malkholm (2018) recounts that TFP growth has been negative in the past seven years, whereas the 15 years leading up to 2000 were positive, and because of this, the choice of historical time period can significantly affect the X-factor. To this point, Meitzen et al. (2017) advise that with declining productivity growth, a backward-looking X-factor may overestimate an industry's capabilities going forward and instead recommend using forward looking X-factors, which summarize the productivity growth differential that would occur if industry suppliers, on average, operate efficiently over the performance-based regulation term.[1]

In the literature, productivity differences among electricity distribution firms can be driven by a variety of factors, including energy density, customer density, network density, peak demand, and the customer mix. For example, Lawrence and Diewert (2004), in their study for the New Zealand Commerce Commission, recommend a three variable output specification based on energy throughput, system capacity, and the number of customers to incorporate important density variables that drive distributors' costs. In addition, the changing nature of investment in the electric industry needs to be reflected by the output trends measured by the output index; for example, investments in advanced metering infrastructure or energy efficiency may not necessarily lead to an increase in outputs. To address these concerns, many TFP studies use a

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[1] See Dr. Weisman's discussion of forward looking X-factors, p. 99 in the pdf, https://www2.auc.ab.ca/h007/Proceeding20414/ProceedingDocuments/EDTINextGenerationPBRPlanSubmission_0076.pdf (accessed 6/13/2022).

mix of output measures (number of customers, line miles, peak usage, etc.) in addition to the traditional output measure (kWh) (Weisman 2018).

With respect to the measurement of capital and other expenses, Joskow (2008) summarizes the need for incentive regulation to have "… a good accounting system for capital and operating costs, cost reporting protocols, data collection and reporting requirements for dimensions of performance other than costs. Capital cost accounting rules are necessary, a rate base for capital must still be defined, depreciation rates specified, and an allowed rate of return on capital determined" (Joskow 2008).

However, with an appropriately designed performance-based regulation program, it may be challenging to recover capital expenditures—it is increasingly common for supplemental capital factor (K), a capital tracker—to be included in the performance-based regulation formula (Weisman 2018). Supplemental capital can lead to overall increases in prices or revenues that exceed inflation as these factors add on to the performance-based regulation plan, as the effective X-factor with a supplemental capital mechanism is $X' = X - K$ (Meitzen et al. 2017). Additionally, supplemental capital mechanisms typically retain elements of cost-of-service regulation, weakening the incentive design of the performance-based regulation plan. Because of the impact on incentives, a performance-based regulation plan with an effective X-factor that accommodates both capital and operations and maintenance (O&M) expenditures is desirable as it encourages a utility to optimize its resources across all inputs (avoiding inefficient substitution between labor and capital, for example). These factors lead Meitzen et al. (2017) to conclude that in the short to medium term, X-factors are likely to be negative, and performance-based regulation plans will require some form of supplemental capital. Designing superior incentives for supplemental capital plans (such as Alberta's K-bar capital mechanism) is also desirable (compared to cost-of-service based capital trackers).

Overall, performance-based regulation can provide firms with incentives to improve their operational efficiency if the firm is the residual claimant for any efficiency gains. Although firms are better informed than regulators about their costs and demand, with the right incentive mechanisms, a regulated firm can be led to maximize society's objectives (be it efficiency or other objectives) (Acton and Vogelsang 1989).

5.0 Importance of Potential Bias for TFP Indexing Method Variables

Although TFP growth estimates from TFP studies can vary widely based on underlying methodologies and assumptions, recommendations to address these potential biases are summarized below.

Table 6. Factors that may Bias TFP and Recommendations

TFP Variable	Variable Choice	Potential Bias	Recommendations
TFP Data	Quality of data available for the sample of selected firms and their input and output data.	Index methods are sensitive to measurement error. The direction and magnitude of the bias will depend on the underlying measurement error.	Publicly available, standardized data (such as those datasets available from FERC or other government agencies) are desirable. Assumptions with respect to data source selection should be documented. Any changes to the data should be documented. If measurement error is a significant concern, econometric approaches to TFP are desirable.
Inflation Indicator	Industry-specific or macroeconomic inflation indicator.	With a macroeconomic inflation indicator, if the input price trend of the economy rises more rapidly than that of the electric industry, the X-factor will be larger, slowing price or revenue growth.	When a macroeconomic inflation index such as the GDPPI is used to measure inflation, there is an additional term known as the input price or inflation differential that must also be estimated.
Length of Study	The X-factor can be calibrated to reflect short- or long-run trends depending on the length of the study.	Short-run trends can be more volatile due to input price or demand fluctuations; long-run trends can smooth these effects. If there is input price volatility, basing the X-factor on long-run trends can cause financial distress for utilities.	The length of the study should be long enough to smooth out volatility in outputs and costs, but reflective of the growth trend that is likely to occur during the PBR period. If it is believed that long-term growth periods are unstable, statistical tests can be used to determine if a structural break has occurred.
Sample Selection	Number and characteristics of included utilities.	In North America, the X-factor is commonly determined based on the productivity growth of a representative sample of firms that	When the TFP metric is productivity growth, heterogeneity across firms largely vanishes, and it is advisable to use the largest possible sample of firms.

TFP Variable	Variable Choice	Potential Bias	Recommendations
		constitutes the electric industry. If the productivity trends are dominated by a handful of utilities, TFP may be biased.	If there is reason to believe that heterogeneity persists, a sample can be restricted to more comparable firms; however, care must be taken to account for exogenous factors that drive productivity differences across firms. For example, firms should face similar productivity growth drivers, such as external business conditions. TFP can be calculated on different sub-sections of samples to understand the impact of particular sample choices.
Output	Measure of output.	Different output measures (such as volume growth or customer count) can cause differences in TFP, with the direction and magnitude of the bias depending on the trend captured by the output measure. For example, volume growth can increase revenues more than costs if volumetric charges are high, creating a positive bias in TFP. Alternatively, volume growth can be slowed by conservation and demand management programs, creating a negative bias in TFP.	Output indexes can consist of more than one output measure to incorporate both customer- and sales-density variables for measuring output for TFP analysis. Many TFP studies use a mix of output measures (number of customers, line miles, peak usage, etc.) in addition to the traditional output measure (kWh) to address these and other changing output trends in the electricity industry. Sensitivity analyses can be performed to assess the sensitivity of TFP growth to various output measures.
Input	Measure of labor.	Most debate over labor measurement is over accurate measurement of labor quantity (i.e., FTEs) or selection of labor price indexes.	Methods should be transparent and replicable.
Input	Measure of materials and services.	Most debate over materials and services is over which expense categories are included or excluded, as well as appropriate price indexes.	Methods should be transparent and replicable. Sensitivity analyses can be performed over inclusion or exclusion of various expenses.

TFP Variable	Variable Choice	Potential Bias	Recommendations
Capital	Choice of benchmark year.	Measurement error in starting capital cost and quantity can create positive or negative bias in TFP estimates.	Benchmark year should allow for many years of plant additions to minimize measurement error.
Capital	Gross or net value of plant in the benchmark year.	Downward bias in TFP trend if net plant value underestimates capital quantity.	Both methods have been used in TFP analyses. The gross capital stock model is appropriate for one-hoss-shay depreciation assumption and the net capital stock model for the geometric decay depreciation assumption (see Diewert and Lawrence 2000). However, existing TFP studies do not always align with the literature in their choice of gross or net plant value. Sensitivity analyses can be performed to determine impacts to TFP from using gross or net value of plant.
Capital	Depreciation method.	Different depreciation methods can result in different capital quantity and price valuations. All three methods (straight-line, one hoss shay, geometric decay) are utilized in TFP studies. The one-hoss-shay method is more sensitive to the useful life of the asset than the geometric decay assumption;[1] however Diewert and Lawrence (2000) found differences in average TFP growth rates from using the three different depreciation assumptions were	Depreciation assumption should best reflect the underlying depreciation profile of the asset. Capital quantity and price indexes should be consistent (i.e., reflect the same depreciation assumptions). Sensitivity analyses can be performed to determine impacts to TFP from using different depreciation assumptions.

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[1] Depreciation is determined entirely by the useful life of the asset with the one-hoss-shay assumption, as discussed in the Depreciation Profiles section of this paper.

TFP Variable	Variable Choice	Potential Bias	Recommendations
		small. However, because the share of capital tends to be large in electricity sector TFP studies, differences in capital valuation may be important to overall TFP.	
Index Weights	Revenue or cost share.	Revenue or cost shares are common and inferred by using econometric models if specific prices are not available. Inaccurate weights can cause changes in output or input indexes that will affect TFP measures.	The choice of revenue or cost share depends on the output or input variable chosen. For example, volume (MWh) as an output measure is typically weighted by its revenue share from customer sales, whereas the number of customers or peak demand is typically weighted by an econometrically inferred cost share. Methodologies for determining revenue or cost shares should be clearly documented and make sense based on the data used to determine the shares.
Index Weights	Chained or multilateral.	Chain-weighted or multilateral index weights are common in TFP studies. The choice of chained or multilateral index can affect TFP as both cost shares and relative growth are computed differently. Chain-weighted index weights are calculated for consecutive periods, whereas multilateral indexes are computed relative to the average firm (see Equation (10)).	With TFP growth either method is appropriate. With TFP levels, only the multilateral method is appropriate. Sensitivity analyses can be performed to assess the sensitivity of TFP growth to various index weighting procedures.
TFP Trends	Arithmetic or weighted average.	Methods to average the TFP trends vary, for example, weights can be a simple arithmetic average or more weight can be given to more similar	Sensitivity analyses can be performed to determine impacts to TFP from different weighting methods.

TFP Variable	Variable Choice	Potential Bias	Recommendations
		firms or more recent years.	
Supplemental Capital	Capital tracker.	Although not a potential bias for TFP, capital trackers can weaken incentives for capex containment.	Consider the “effective” X-factor: $X' = X - K$ Consider designing superior incentives for supplemental capital plans.

6.0 Conclusion and Next Steps

Performance-based regulation was introduced to improve upon and as an alternative to cost-of-service regulation, providing utilities with incentives similar to those faced by companies operating in competitive markets, and encouraging them to focus on operational efficiency and cost reductions. Although there are many approaches to strengthen utility performance incentives, in this review, we primarily focus on total factor productivity (TFP) studies that inform the X-factor in price or revenue cap regulation, providing an overview of the economic principles that underly the X-factor, common methods for estimating TFP, including index number methods, approaches to measuring outputs and inputs when using index number methods, as well as a review of lessons learned from the TFP literature and recent performance based regulation cases.

Although TFP measurement does not have a one-size-fits-all approach, as varied assumptions and appropriateness of methodologies will depend on the unique circumstances of the individual utilities, we also provide a summary of factors which may bias TFP studies and recommendations to address potential biases. Importantly, we include recommendations for addressing concerns of bias in typically controversial elements of a TFP study, including the length of the study, sample selection, output measurement, and capital measurement. Some key takeaways are that study methodologies and assumptions should be transparent enough that the study could be reproduced, and sensitivity analysis of key assumptions can be undertaken to show the sensitivity of TFP to changing those key assumptions.

In future work, we will utilize the methodologies, assumptions, best practices, and potential biases outlined in this review to perform a critical review of past TFP studies from two X-factor proposals in Massachusetts [Eversource (D.P.U. 17-05) and National Grid (D.P.U. 18-150)]. We will evaluate the method and assumptions chosen, provide an objective summary of the benefits and drawbacks of that method, provide recommendations for alternative data, methods, and assumptions that would improve the accuracy or reasonableness of the analysis, and provide additional criteria to consider for evaluation of future TFP studies.

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Appendix A – Supplementary Material

A.1 Non-Parametric Frontier Methods: Data Envelopment Analysis

The data envelopment analysis (DEA) starts with Charnes et al. (1978), which is based on work by Farrell (1957). DEA is a linear programming technique that identifies the most efficient (or best practice) firms by fitting a frontier over the sample of firms. Less efficient firms are measured relative to the frontier. Technically, this method estimates efficiency by measuring the ratio of total output produced to total inputs employed by each firm, comparing this ratio to other firms in the sample to estimate relative efficiency (Abbott 2005).

Van Biesebroeck (2007) explains that in practice, efficiency, θ, of a unit (firm-year) is defined as the ratio of a linear combination of outputs (Q) to a linear combination of inputs $(L,\ K)$, where weights on output (v_q) and inputs $(u_l,\ u_k)$ are chosen to maximize efficiency for the unit under consideration (here, the unit is firm-year $i = 1$). If another unit produces more output with the same amount of input using identical input weights, unit 1 is considered dominated. For the case of one output, a linear-programming problem is solved separately for each unit $(i = 1 \ldots N)$.

$$\begin{gathered}\max_{v_q,u_l,u_k} \theta_1 = \frac{v_q Q_1 + v^*}{u_l L_1 + u_k K_1} \\ \text{subject to} \frac{v_q Q_i + v^*}{u_l L_i + u_k K_i} \leq 1 \quad i = 1 \ldots N \\ v_q, u_l + u_k > 0, u_l, u_k \geq 0 \\ v^* \geq 0 \quad (v^* = 0 \text{ for constant returns to scale})\end{gathered} \qquad \text{A.1}$$

Weights have sign restrictions as noted above, and the efficiency of each firm cannot exceed 100% when using the same weighting scheme. Efficiency can be interpreted as the productivity difference between unit i and the most efficient unit, $\theta_i = A_i/A_{max}$. Compared to the index number methodology, DEA estimates

$$lnA_{it} - \overline{lnA_t} = ln\theta_{it} - \frac{1}{N}\sum_{j=1}^{N_t} ln\,\theta_{jt}. \qquad \text{A.2}$$

The unit of observation is each firm-year. DEA is the preferred estimator if production technology is likely to vary across firms or economies of scale are not constant—for example, if firms at very different stages in their lifecycles are being pooled because DEA does not require any specification of production technology (Van Biesebroeck 2007). DEA also has the benefit of being able to separate total factor productivity (TFP) into its component parts of allocative efficiency (due to efficiency in resource use) or technical efficiency (due to technological change), which can help identify potential efficiency improvements. DEA can also be used to estimate the change in productivity of individual firms over time. However, DEA is sensitive to the weights chosen and the method is sensitive to outliers. As each unit's efficiency is derived from a comparison to the sample of units, measurement error can also affect all estimates. DEA is widely used in academic studies of TFP but rarely used in performance-based regulation due to its perceived complexity (Frayer et al. 2016).

A.2 Parametric Frontier Methods: Stochastic Frontier Methods

Similar to DEA, stochastic frontier methods estimate a firm's efficiency by first constructing a production frontier from best practice firms.[1] A key differentiation with this method is the assumptions used to separate the distribution of the unobserved productivity parameter (ω_{it}, in the econometric specification of production) from the random error. The term, ω_{it}, is interpreted as the inefficiency of firm i at time t relative to the most efficient (best practice) firms in the sample.

The productivity term ω_{it} is modeled using assumptions about the distribution of productivity in the sample of firms. For example, Battese and Coelli (1992) assume the productivity of each firm can vary over time and is drawn from a truncated normal distribution with mean, γ, and variance, σ^2. With these assumptions, the productivity parameter can be modeled as

$$\omega_{it} = e^{-\eta(t-T)}\omega_i \qquad \text{with } \omega_i \sim N^+(\gamma, \sigma^2) \tag{A.3}$$

Where the econometrician has data on a sample of *N* firms over T time periods. Efficiency increases (decreases) over time if η is positive (negative) (Battese and Coelli 1992; Van Biesebroeck 2007). The parameter ω is usually estimated using maximum likelihood methods (see Coelli [1992] and Greene [2010]) where technical efficiency is calculated as

$$TE_{it} = E(e^{-\omega_{it}} | \omega_{it} + \epsilon_{it}). \tag{A.4}$$

Benefits of the stochastic frontier method are that it produces accurate productivity levels if output is measured accurately, firms share the same technology, and productivity differences among firms do not change over time (Van Biesebroeck 2007). Drawbacks of the methodology are that it does require specification of the production technology (typically Cobb-Douglas or translog production functions) and it is not typically used in ratemaking cases but often used in academic studies.[2]

A.3 Two Superlative Indexes with Multiple Outputs and Inputs in Practice: The Fisher Index and the Törnqvist Index

A.3.1 Fisher Index

Mathematically, the Fisher ideal output index is given by

$$Q_t^F = \left(\frac{\sum_{m=1}^{M} p_{mB}\, Q_{mt}}{\sum_{n=1}^{M} p_{nB}\, Q_{nB}} * \frac{\sum_{m=1}^{M} p_{mt}\, Q_{mt}}{\sum_{n=1}^{M} p_{nt}\, Q_{nB}} \right)^{\frac{1}{2}} \tag{A.5}$$

where Q_t^F is the Fisher ideal output index, p is the price of output, and Q is the output quantity. The subscripts m or *n* refer to diverse outputs. There is a total of M outputs. The subscript t refers to the observation year, and the subscript B refers to the base year observation.

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[1] See Aigner et al. (1977) for the stochastic frontier model that is the foundation of this stream of the literature.

[2] See Kumbhakar et al. (2020) for a survey of the literature.

Similarly, the Fisher ideal input index can be mathematically represented as

$$I_t^F = \left(\frac{\sum_{m=1}^{N} w_{mB} \, X_{mt}}{\sum_{n=1}^{N} w_{nB} \, X_{nB}} * \frac{\sum_{m=1}^{N} w_{mt} \, X_{mt}}{\sum_{n=1}^{N} w_{nt} \, X_{nB}} \right)^{\frac{1}{2}} \qquad \text{A.6}$$

Where I_t^F is the Fisher ideal output index, w is the input price, and X is the input quantity. The subscripts m or n refer to diverse inputs, and there is a total of N inputs. The Fisher ideal TFP index is given by

$$TFP_t^F = \frac{Q_t^F}{I_t^F}. \qquad \text{A.7}$$

While the Fisher index is appropriate for comparing rates of change of productivity between firms or over time, it does not allow for comparisons in absolute levels of productivity between firms (Lawrence and Diewert, 2004). Instead, to compare both levels and growth rates of productivity, practitioners use the Törnqvist index.

A.3.2 Törnqvist Index

In practice, to compare two observations denoted as *i* and j, the Tornqvist index for m diverse outputs (Q) and n diverse inputs (X, which could represent labor, capital, or other relevant inputs) is

$$\begin{aligned} ln\left(\frac{TFP_i}{TFP_j}\right) = & \sum_m \frac{(R_{m,i} + \overline{R_m})}{2}\left(lnQ_{m,i} - \overline{lnQ_m}\right) - \sum_m \frac{(R_{m,j} + \overline{R_m})}{2}\left(lnQ_{m,j} - \overline{lnQ_m}\right) \\ & - \sum_n \frac{(S_{n,i} + \overline{S_n})}{2}\left(lnX_{n,i} - \overline{lnX_n}\right) + \sum_n \frac{(S_{n,j} + \overline{S_n})}{2}\left(lnX_{n,j} - \overline{lnX_n}\right), \end{aligned} \qquad \text{A.8}$$

where R_m and S_n are output and input weights. The average revenue ($\overline{R_m}$) or cost share ($\overline{S_n}$) is averaged over all utilities and time periods. Q_m and X_n are output and input quantities. The average log of output is given by $\overline{lnQ_m}$ and the average log of input is given by $\overline{lnX_n}$. Equation A.8 gives the proportional change in TFP between two observations. Lawrence and Diewert (2004) discuss that in practice the Törnqvist index is formed by setting an observation equal to one (usually the first observation in the database) and multiplying through by the proportional changes between all subsequent observations in the database. Because the Törnqvist index is independent of the utility or year used as the base observation, it is transitive, essentially comparing each observation to a hypothetical average firm. See Lawrence and Diewert (2004) for further reading.

A.4 Input Index Quantity and Price Measurement

Table A.1. Common Assumptions Used in Measuring Input Quantities and Prices

Study	Price/Rev. Cap	Input	Quantity Measurement	Price Measurement	Weights and Index
"2018 – 2022 Performance-Based Regulation Plans for Alberta Electric and Gas Distribution Utilities (Errata to Decision 20414-D01-2016)"[1] *Note that this study primarily leveraged the Makholm and Ros (2010) study methodology described below, with modifications as noted. See written evidence of Dr. Brown and	Rate (price) cap for electric dist. companies; revenue-per-customer cap for gas dist. Utilities.	Labor	Labor is based on an estimate of full-time equivalent employees (number of full-time equivalent employees [FTEs] + ½ the number of part-time employees). Because FERC Form 1 no longer contained employee data after 2001, growth of the U.S. BLS series of wages and salaries in the utility sector is used to obtain a constant dollar estimate of labor input. *A correction is made here to extend the series using constant dollar total salaries (not constant dollar distribution salaries), then multiplying by the ratio of distribution salaries to total salaries to obtain distribution employee quantity. This error was identified by Meitzen (2016).	Distribution salaries from FERC Form 1.	Multilateral Törnqvist Index.

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[1] https://www2.auc.ab.ca/h007/Proceeding20414/ProceedingDocuments/20414-D01-2016Errata2018-2022PBRPlansfor_0712.pdf (accessed 6/6/2022). https://www.regulatorylawchambers.ca/blog/2018/12/10/decision-20414-d01-2016-re-2018-2022-pbr-plans-for-alberta-electric-and-gas-distribution-utilities (accessed 6/6/2022).

Study	Price/Rev. Cap	Input	Quantity Measurement	Price Measurement	Weights and Index
Dr. Carpenter. (Brown and Carpenter, 2016)[1]					
		Materials	The quantity of materials is obtained by deflating the cost of materials by the GDPPI.	The cost of materials is residually obtained by subtracting distribution labor costs from distribution operations and maintenance cost.	
		Capital	Capital is computed using a perpetual inventory "one-hoss-shay" method. Perpetual inventory method uses the 1964 book value of distribution plant in service, the Handy-Whitman index for distribution plant, annual additions to plant, and retirements from the plant. The benchmark capital stock quantity is calculated by applying a trailing weighted average of Handy-Whitman prices to the 1964 book value of plant. Additions to plant are deflated by the current year's Handy-Whitman index value. Retirements are deflated by the Handy-Whitman index value, lagged by the assumed average lifetime of distribution plant.	Capital rental price is the dual to the one-hoss-shay capital quantity. Capital rental price is calculated using data on annual yields to impute expected future rates of return on investment Brown and Carpenter (2016) were not able to obtain the same data on credit ratings and bond yields as used by Makholm and Ros (2010) but obtained similar data from another provider.	

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[1] https://www2.auc.ab.ca/h007/Proceeding20414/ProceedingDocuments/AppendixA-BrattleWrittenEvidence_0059.pdf (accessed 6/14/2022). https://www2.auc.ab.ca/h007/Proceeding20414/ProceedingDocuments/2016-05-27-Brattlereplyevidenceonnextgen_0397.pdf (accessed 6/14/2022).

Study	Price/Rev. Cap	Input	Quantity Measurement	Price Measurement	Weights and Index
Determination of the Second-Generation X Factor for the AUC Price Cap Plan for Alberta Electric Distribution Companies (Meitzen, 2016) *Note that this study primarily leveraged the Makholm and Ros (2010) study methodology described below, with modifications as noted.[1]	Rate (price) cap for electric dist. companies; revenue-per-customer cap for gas dist. Utilities.	Labor	Labor is based on an estimate of FTEs (number of full-time employees + ½ the number of part-time employees). FTEs are multiplied by the ratio of distribution salaries to total salaries to obtain distribution employee quantity. Because FERC Form 1 no longer contains employee data after 2001, growth of the U.S. BLS series of wages and salaries in the utility sector is used to obtain a constant dollar estimate of labor input. *A correction is made here to extend the series using constant dollar total salaries (not constant dollar distribution salaries), then multiplying by the ratio of distribution salaries to total salaries to obtain distribution employee quantity.	Distribution salaries from FERC Form 1.	Multilateral Törnqvist Index.
		Materials	The quantity of materials is obtained by deflating the cost of materials by the GDPPI.	The cost of materials is residually obtained by subtracting distribution labor costs from distribution operations and maintenance cost.	
		Capital	Capital is computed using a perpetual inventory "one-hoss-shay" method.	Capital rental price is the dual to the one-hoss-shay capital quantity.	

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[1] https://www2.auc.ab.ca/h007/Proceeding20414/ProceedingDocuments/EDTINextGenerationPBRPlanSubmission_0076.pdf (accessed

Study	Price/Rev. Cap	Input	Quantity Measurement	Price Measurement	Weights and Index
			Perpetual inventory method uses the 1964 book value of distribution plant in service, the Handy-Whitman index for distribution plant, annual additions to plant, and retirements from the plant. The benchmark capital stock quantity is calculated by applying a trailing weighted average of Handy-Whitman prices to the 1964 book value of plant. Additions to plant are deflated by the current year's Handy-Whitman index value. Retirements are deflated by the Handy-Whitman index value, lagged by the assumed average lifetime of distribution plant.	Capital rental price is calculated using data on annual yields to impute expected future rates of return on investment. Meitzen (2016) used Makholm and Ros (2010) 2009 capital rental price data due to lack of data availability.	
2018 – 2022 Performance-Based Regulation Plans for Alberta Electric and Gas Distribution Utilities (Errata to Decision 20414-D01-2016).[1]	Rate (price) cap for electric dist. companies; revenue-per-customer cap for gas dist. Utilities.	Labor	Ratio of salary and wage expenses to a regionalized salary and wage labor price index.	The cost of labor was determined as O&M salaries and wages and pensions and other benefits.	Chain-weighted Törnqvist Index.
		Materials and Services (M&S)	Ratio of expenses for these inputs to an M&S price index developed by PEG from produce price indexes from the U.S. BLS. Applicable expense included those reported for power distribution and	The cost of M&S was determined as applicable O&M expenses net of labor costs.	

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[1] https://www2.auc.ab.ca/h007/Proceeding20414/ProceedingDocuments/20414-D01-2016Errata2018-2022PBRPlansfor_0712.pdf (accessed 6/6/2022). https://www.regulatorylawchambers.ca/blog/2018/12/10/decision-20414-d01-2016-re-2018-2022-pbr-plans-for-alberta-electric-and-gas-distribution-utilities (accessed 6/6/2022).

Study	Price/Rev. Cap	Input	Quantity Measurement	Price Measurement	Weights and Index
Next Generation PBR for Alberta Energy Distributors. (Lowry, 2016)[1]			meter reading, plus a sensible share of administrative and general expenses (exclusive of those for pension and benefits). *Excluded reported costs of any gas services. Excluded other costs that were unlikely to be indexed (purchased power, power transmission by others, franchise fees, customer service and information, sales, and most customer account functions).		
		Capital	Capital was separated into distribution plant and general plant. Capital quantity was constructed using inflation-adjusted data on the value of the utility plant. A geometric decay depreciation assumption was used, but results were comparable with a cost-of-service approach. The benchmark value was constructed based on the net value of the plant in 1964, adjusted for inflation. Equivalent to the book value of the plant divided by an average of the values of an index of utility construction cost for a period ending in the benchmark year. The following formula was used to compute capital quantity for subsequent values:	Capital cost is the sum of depreciation expenses, a return on the value of net plant, and taxes $WKS_t = \left[\frac{CK_{j,t}^{Taxes}}{XK_{j,t-1}}\right] dWKA_{j,t} + WKA_{j,t-1}\left[r_t - \frac{WKA_{j,t} - WKA_{j,t-1}}{WKA_{j,t-1}}\right]$ Where the first term indicates taxes and franchise fees, the second term, the cost of depreciation, and the third term, the real rate of capital. Prices were	

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[1] https://www2.auc.ab.ca/h007/Proceeding20414/ProceedingDocuments/CCAEvidenceofPEG_0084.pdf (accessed 6/14/2022).

Study	Price/Rev. Cap	Input	Quantity Measurement	Price Measurement	Weights and Index
			$XK_{j,t} = (1-d)XK_{j,t-1} + VI_{j,t}/WKA_{j,t}$ where *XK* is capital quantity, *d* is the economic depreciation rate, *VI* is gross additions to plant, and *WKA* is the construction cost index; t indexes time.	smoothed to reduce capital cost volatility.	
Productivity and Benchmarking Research in Support of Incentive Rate Setting in Ontario. (Kaufmann et al., 2013)[1]	Price cap	Operation, Maint. & Admin. (OM&A)	OM&A quantities were estimated as the ratio of distribution OM&A expenses to an index of OM&A prices.	A labor price index, the average weekly earnings for all laborers in Ontario, was used for labor prices. Non-labor prices were measured with GDP-IPI, which is an index that applies to all final domestic demand in Canada.	Törnqvist index. Data on share of labor and non-labor expenses for OM&A are confidential but were previously estimated as 70% labor by staff at the Ontario Energy Board. This estimate was used by PEG. The weight applied to the capital input price index is calculated as the electricity distributors' capital cost divided by the total cost measure.
		Capital	Benchmark capital stock was based on gross plant value in 1989. Benchmark capital stock was deflated by a weighted average of capital prices preceding the 1989 benchmark capital value. Subsequent values of the capital quantity index were computed via the perpetual inventory equation: $XK_t = (1-d)XK_{t-1} + VI_t/WKA_t$ where *XK* is capital quantity, *d* is the economic depreciation rate, *VI* is gross additions to plant, and *WKA* is	Capital service price is estimated based on both the depreciation and the rate of return on capital: $WKS_t = d \times WKA_t + WKA_{t-1} \times r_t$ Where d is the rate of depreciation, r is the rate of return on capital, and WKA is the asset price index.	

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[1] https://www.oeb.ca/oeb/_Documents/EB-2010-0379/EB-2010-0379_Final_PEG_Report_20131111.pdf (accessed 6/9/2022).

Study	Price/Rev. Cap	Input	Quantity Measurement	Price Measurement	Weights and Index
			the construction cost index; t indexes time.	A geometric depreciation rate was assumed.	
2018 – 2022 Performance-Based Regulation Plans for Alberta Electric and Gas Distribution Utilities (Errata to Decision 20414-D01-2016).[1] "Total Factor Productivity Study for Use in AUC Proceeding 566 – Rate Regulation Initiative," December 30, 2010 (Makholm and Ros, 2010).[2] NERA, "Update, Reply and PBR Plan Review for AUC Proceeding 566 – Rate Regulation	Rate (price) cap for electric dist. companies; revenue-per-customer cap for gas dist. Utilities.	Labor	Labor is based on an estimate of full-time equivalent employees (number of FTEs + ½ the number of part-time employees). Because FERC Form 1 does not contain employee data after 2001, growth of the U.S. BLS series of wages and salaries in the utility sector is used to obtain a constant dollar estimate of labor input.	Distribution salaries from FERC Form 1.	Multilateral Törnqvist Index.
		MRS	The quantity of materials is obtained by deflating the cost of materials by the GDPPI.	The cost of materials is residually obtained by subtracting distribution labor costs from distribution operations and maintenance cost.	
		Capital	Capital is computed using a perpetual inventory "one-hoss-shay" method. Perpetual inventory method uses the 1964 book value of distribution plant in service, the Handy-Whitman index for distribution plant, annual additions to plant, and retirements from the plant.	Capital rental price is the dual to the one-hoss-shay capital quantity. Capital rental price is calculated using data on annual yields to impute expected future rates of return on investment.	

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[1] https://www2.auc.ab.ca/h007/Proceeding20414/ProceedingDocuments/20414-D01-2016Errata2018-2022PBRPlansfor_0712.pdf accessed 6/6/2022). https://www.regulatorylawchambers.ca/blog/2018/12/10/decision-20414-d01-2016-re-2018-2022-pbr-plans-for-alberta-electric-and-gas-distribution-utilities (accessed 6/6/2022).

[2] https://www2.auc.ab.ca/h002/Proceeding566/ProceedingDocuments/1a_ID566%20N_0204.pdf (accessed 6/22/2022).

Study	Price/Rev. Cap	Input	Quantity Measurement	Price Measurement	Weights and Index
Initiative," February 22, 2012 (Makholm and Ros, 2012).[1]			The benchmark capital stock quantity is calculated by applying a trailing weighted average of Handy-Whitman prices to the 1964 book value of plant. Additions to plant are deflated by the current year's Handy-Whitman index value. Retirements are deflated by the Handy-Whitman index value, lagged by the assumed average lifetime of distribution plant.		
Total Factor Productivity and Performance-Based Ratemaking for Electricity and Gas Distribution (Makholm et al., 2010).	Other: TFP analysis for the U.S. Electric Industry (1972 – 2009).	Labor	Number of employees. Specifically, labor quantity is comprised of the number of full-time employees and 50% of the part-time and temporary employees to obtain FTEs.	The price of labor is calculated by dividing Direct Payroll to Electric Distribution by FTEs attributed to Distribution.	The aggregate input index was comprised of labor, capital and materials, rents, and services indexes. Labor cost shares were from the FERC account Direct Payroll to Electric Distribution. MRS cost shares were from the MRS expense (FERC account Total Distribution Operation and Maintenance
		Materials, Rents and Services (MRS)	MRS quantity is determined by a two-step process: (1) MRS expenses are calculated as the difference between operating expenses and labor expenses (from FERC Form 1). (2) MRS expense is deflated by dividing MRS expense by the GDPPI to obtain a measure of the MRS quantity input.	The MRS price is measured by the U.S. GDPPI.	
		Capital	The book value of the plant and the Handy-Whitman Index (*HW*) is used	Capital prices (P) are based on the	

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[1] https://www2.auc.ab.ca/h002/Proceeding566/ProceedingDocuments/Second%20Rep_1425.pdf (accessed 6/22/2022).

Study	Price/Rev. Cap	Input	Quantity Measurement	Price Measurement	Weights and Index
			to compute capital quantity for the benchmark year ($K_{benchmark}$). $K_{benchmark} = \frac{book\ value\ of\ plant_{benchmark}}{\sum_{i=1}^{20} i \left[\frac{i}{\sum_{i=1}^{20} i}\right] HW_{1994+i}}$ After the benchmark year, capital is added according to the following formula: $K_t = K_{t-1} + \frac{gross\ additions\ to\ plant}{HW_t} - \frac{retirements_t}{HW_{t-s}}$ where *s* is the useful life of the asset. The “one-hoss-shay” depreciation pattern is used for depreciation of capital.	acquisition price of new capital and the present value of all of its future services: $P_{k,t} = \left(\frac{1-k-uz}{1-u}\right)(r - i)\left[1 - \left(\frac{1+i}{1+r}\right)^{s}\right]^{-1} HW_{t-1}$ Where: *k* = investment tax credit rate, *u* = corporate profits tax rate, *z* = present value of depreciation deduction on new investment, *r* = cost of capital, *i* = expected inflation rate over the asset’s lifetime, and HW_{t-1} = Handy-Whitman’s asset price in the prior year.	Expenses [excluding salary expenses]) minus Direct Payroll to Electric Distribution. Capital share is determined by the quantity of capital times the price of capital. Törnqvist index method is used for TFP measurement.

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